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mathematical models in educational planning

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PREFACE

In 1966 the OECD Committee for Scientific and Technical Personnel embarked upon an experimental programme to examine and evaluate applications of systems analysis, operational research and related techniques to practical educational planning.

Preliminary investigations revealed a considerable interest in the possibility of using such techniques by educational planners and some confidence amongst systems analysts, operational researchers and econometricians that they could make a useful contribution in this field. One important area in which theoretical work has reached a fairly advanced stage in a number of OECD Member countries is the construction of quantitative models of national educational systems which can be used as a basis for forecasting or planning future student numbers in accordance with economic and social needs. This topic was the subject at a meeting of model builders and planners organised by the OECD in March 1966. The general conclusion was that these comprehensive quantitative approaches to global educational planning problems hold out considerable promise for improving the reliability of forecasts and the efficacy of the information base for planning, and that they can ultimately be developed into general planning tools with the aid of which the implications of different policies can be evaluated. The present volume contains the papers which were presented at the meeting together with an introductory chapter by professor J.R.N. Stone of Cambridge University placing the models and the techniques used in the general context of economic and social planning.

Alexander King
Director for Scientific Affairs.

PART I : INTRODUCTION

A VIEW OF THE CONFERENCE

by Richard Stone

1. The setting

The papers in this volume examine from various points of view the possibilities of applying a number of related techniques, such as mathematical model building, simulation, systematic control theory, in short, systems analysis, to the problems of educational planning. Even in the countries where it can be said to exist at all, educational planning is a very recent development arising from the transformation of the scale of educational endeavour, which in many countries of Europe amounts to little less than an educational revolution. The diverse and often conflicting aims of education, the complex structure of the educational system itself and the great cost of educational programmes mean that if desired results are to be achieved efficiently the educational system must be looked at as a whole. Day-to-day administrative decisions must somehow be coordinated within the framework of a consistent policy. And such a policy cannot be shaped unless a longer and broader view is taken of the functioning of the educational system in relation to social and economic needs on the one hand and human and financial resources on the other.

Those who attended the conference at which these papers were discussed combined the belief that the methods described hold great promise for the future with the recognition that much more work is needed before they can be said to have passed from the pilot to the operational stage. In this first chapter, therefore, an attempt is made to summarise the general consensus of opinion which emerged clearly from the discussions, partly to show research workers the possibilities of an important new field of research and partly to show educators, educational administrators and educational planners the possibilities of new methods which only now are beginning to be applied to educational problems.

2. Systems and systems analysis

According to Webster's dictionary the most general meaning of the word 'system' is: an aggregation or assemblage of objects united by some form of regular interaction or interdependence; a natural combination, or organisation of part to part, conceived as formed by a process of growth or as due to the nature of the objects connected; an organic whole. We need not go beyond some such description as this in order to understand the meaning of the term the 'educational system'.

The study of any system can be divided into a number of distinct stages. First, we must isolate and define the system itself according to the purpose of our study; thus we may speak of a system of railways, but for some purposes this may be regarded simply as part of the transport system in general. Second, we must describe the system in such a way that we can analyse it and so be in a position to draw conclusions about those aspects of it that interest us. Third, having specified the variables that enter into our desciption, we must collect information about them so as to secure the data for analysis. Fourth, we must formulate the relationships that we think connect the variables and we must estimate the parameters that enter into these relationships.

Up to this point we have a model of the system ready for application. But, fifth, if we want to use this model to help us to plan, we must specify our aims in terms of the variables we have used to describe the system. We can then try to discover whether these aims can be realised with the present operating characteristics of the system. If so, is there more than one way to do this, and can there be said to be a 'best' way? If not, how could we modify the system so that the aims could be realised, and is there a 'best' way of doing this?

Finally, we must establish some means of regulating the system so that its performance comes close to our aims. In a physical system we should try to design a control device, like the governor of steam engine, which would enable the system to regulate itself. In more complicated cases, like the present one, such automatic devices are likely to play a limited role and more reliance must be placed on human decisions in controlling the system ; indeed, it must be recognised that there may be features of the system which can be controlled only within limits or cannot be controlled at all with the means of control that are considered acceptable.

This ordered catalogue of steps is useful only for expository purposes. In real life, models, aims and controls interact and, whereever we begin our investigation, we must recognise that at the outset our knowledge of the other steps will be incomplete. We must begin to collect and arrange data without knowing all about our aims which, even if fully specified, would almost certainly have to be modified when we spelt out some of their implications in terms of cost and indirect effects. We must begin to specify aims without being sure initially that they are attainable or even, in the final analysis, desirable . We must begin to regulate the system without having enough information about its operating characteristics to design efficient control devices. What is important is that we should keep all these aspects of the problem in mind and gradually develop an analytical tool for educational decisions which can make systematic use of all relevant data, allows for imperfect information and incorporates a learning mechanism capable of responding promptly and effectively to experience and calculation.

At the conference all these topics were discussed. Let us now look at them in greater detail.

3. Describing the system

The authors who presented numerical descriptions of their countries' educational systems concentrated on formal education in schools, colleges and universities to the exclusion of part-time vocational and professional training undertaken after formal education has ceased. The reason for this concentration was simply that a start has to be made somewhere and that the obvious starting point is formal education. At the same time it was recognised that informal education is important since one aspect of educational planning relates to the needs of the economy, and it was considered necessary to get more information about this aspect. It was emphasised that in advanced countries nowadays the length of life of a particular occupational skill might in many cases be as short as fifteen years, so that retraining in adult life was becoming a matter of necessity for a growing proportion of the labour force. This tendency, it was thought, was likely to react on the nature of formal education, where it would be necessary to place more emphasis on adaptability and general education and less on rigid and specialised education which would tend to freeze the student at a particular stage of a rapidly developing subject.

Having isolated the system of formal education as the subject to study first, the authors divided this system into a number of branches and concentrated on the flow of students through this disaggregated system and on the stocks of students in different parts of it. In addition to a statement of student flows and stocks, it would also be necessary to set out a corresponding statement of economic flows and stocks, namely of the teachers, materials, equipment and buildings necessary to operate the different branches of the system. This double system of flows and stocks, demographic and economic, would provide a quantitative picture of the educational structure as it is. A model based on this structure would help us to estimate the scale of different educational activities implied by the growth of the population or needed to ensure that the future composition of leavers, or 'graduate mix', accorded with the aims set for the system's performance.

This work on student flows, which can be formalised in what may be called a demographic accounting system, was compared with the economic accounting systems for whole economies which have been developed in most countries in the last twenty years. These economic accounting systems have proved an indispensable framework for national economic policies and have been expanded, from small and relatively inaccurate beginnings, in the light of policy needs mediated through theoretical considerations and statistical possibilities. The view was expressed and generally accepted that a complementary system of social accounting for education was an indispensable first step which should be given top priority by educational model builders. Without a well-defined picture, that can easily be kept up to date, of the existing educational structure, it is impossible, it was argued, to consider modifications of this structure on anything but a piece-meal basis, looking at one problem after another without being able to trace their interactions.

The limitations of existing data were discussed and a number of specific problems emerged, some of which call for action on the part of educational statisticians.

First, at present there is a general lack of statistics on flows of students; with few exceptions, flows have to be derived from information about stocks. In many cases a reasonable approximation can be reached by recognising that many possible flows are unlikely to be more than trickles and can therefore be ignored and by making full use of the arithmetical and accounting identities which a systematic organisation of the data reveals. However, this is not always so and, since flow statistics enter so largely into descriptive models it would be helpful if a greater statistical effort could be concentrated on them.

Second, it is often found that published information sufficient to build up a detailed picture of student flows is only available for recent years, perhaps only for two or three years at the most. Here the collection of new data cannot help in providing a picture of the past, but research into existing records might enable at least parts of the picture to be filled in. Since a knowledge of the changing structure of education would be useful, this subject might repay investigation by institutes devoted to educational research.

Third, the available statistics are usually highly aggregated: they relate to categories rather than to indivuduals. There is a growing recognition that individual data systems are in many ways desirable, particularly where administrative action is concerned with individuals as well as with categories, as in the case of teachers, and where age, location, qualifications and other characteristics of the individual are at least as significant as the broad class in which the individual is classified. Since individual data systems, for students as well as teachers, cannot be introduced quickly, it was thought that their potential value in educational planning was such that the time had come for an appraisal of their merits and of the problems of instituting them in any country seriously interested in educational planning.

A number of other questions relating to data were discussed in connection with estimating the parameters in the models described at the conference. These questions are easier to understand when something has been said about the kind of relationships that enter into these models, and so a discussion of them will be deferred to section 5 below.

4. Theories

Theories relate to the way in which we propose to connect variables to one another, to our aims and to possible instruments of control. It is convenient to distinguish two kinds of theory, both of which were represented in the papers given to the conference. The first kind starts from a set of observations and asks how these observations can be related. This kind of theory usually takes it for granted that the first attempts at relating the observations will be only approximate and looks to experience to suggest improvements, which may be needed as much in the data and the way they are organised as in the theory itself. Such theories may be called "theories for application".

By contrast, the second kind starts with a problem and asks how in principle it could be solved even if, initially, no data are available. This kind of theory is usually concerned with the character of the solution under fairly general conditions. The problem is usually posed in a highly abstract manner which offers some hope of solution if sufficiently advanced methods are used. Such theories may be called "theories for insight".

The first kind of theory was illustrated by the papers at the conference which were designed to organise information about flows of students. The general idea in this case was that the educational system can be represented as a set of branches through which students flow. We can observe that the students in any branch in one period distribute themselves over branches in the next period in given proportions. If these proportions were fixed, the future activity levels of all the branches would depend partly on the present numbers in the different branches and partly on the new entrants to the system from births and migration. Given the future course of these demographic variables, therefore, the future activity levels of each branch of the system could be calculated and so could the system's future final product, that is the numbers of students leaving the system altogether from one or other of its different branches.

This model is conceptually and algebraically simple, and the problems that have to be solved before it can be applied are mainly the taxonomic and data problems discussed in the preceding section. Its merit, shared with its economic counterpart input-output analysis, which in many respects it resembles, is that it enables a very large body of data to be processed systematically to give quantitative answers to important questions. The answers will be approximate, however, because the relationships of the model are simple and because the information used to estimate its parameters, the transition proportions, are in practice likely to be biased in various ways. In addition, as with economic input-output, there are the problems of changing parameters and of the introduction of new processes and new products.

The second kind of theory was illustrated by a model designed to distribute the fifty-year period from the end of compulsory education to retirement among full-time education, part-time education and full-time earning by reference to the earning prospects which would follow from different distributions. This model, too, has an economic counterpart, in Ramsey's theory of saving, which is concerned with the rate of saving that a community should adopt if it wishes to maximise the satisfaction it can expect to obtain by consuming less or working harder now in order to be able to consume more or work less in the future. Both problems are extremum problems, but whereas Ramsey was able to formulate his so that it was accessible to the classical methods of the calculus of variations, the present problem could not be formulated in this way. It was solved, however by an application of Pontryagin's maximum principle and, in discrete form, would have been accessible to Bellman's technique of dynamic programming. Though formulated and solved at a rather high level of abstraction, the author indicated ways in which many of his assumptions could be relaxed. One of these related to the introduction of different kinds of education into the model. It was suggested in discussion that if the distinction were made between general and vocational education, it would be interesting to see how the model would react to the high obsolescence rate of many specific skills which was mentioned earlier. With these additional features, the model might shed some light on the desirability of a general education, favouring adaptability, in youth, followed by spells of highly specialised education at various stages of adulthood, as opposed to a relatively specialised education in youth followed by little if any further education in adulthood, which is the prevalent pattern today.

As has been said already, the distinction between theories for application and theories for insight is one of convenience: there is no suggestion that the former can give no insight or that the latter cannot be applied to real problems. They are, however superficially very different and it is important to recognise that each has its contribution to make.

In the discussion a number of points were made.

First, there was a considerable similarity in the models of student flows presented to the conference. It was thought that an international organisation could help greatly by providing opportunities for model builders to keep in touch and by encouraging uniformities of treatment where this was desirable. In this way it would be easier to compare the experience of different countries, and results achieved in one country could be replicated in another.

Second models of economic inputs into education were not much discussed, but the link provided by the fact that the educational system produces its own major input, teachers, was emphasised. Economic input output tabulation and analysis offers a pattern which could readily be applied to the educational system. It was thought that more information was desirable on the effect of grants in influencing the choices of students.

Third, the models of student flows divided the educational system up into branches but did not try to get inside the branches and see what was going on there: in other words the educator and the psychologist were left out of the models. This defect would have to be remedied because existing educational techniques could not be taken for granted in planning the great expansion of education that was now beginning to take place.

Fourth, and this point is closely related to the preceding one, more attention should be paid to the concept of productivity in education: an impartial look at existing curricula and their effectiveness is much needed. It was recognised that to do this new concepts and methods of measurement would have to be developed. It was also recognised that many educators associated high productivity with bad education and were generally suspicious of productivity studies. It was thought, however, that the inputs of resources needed by present and prospective educational programmes were so high that the problem could not be neglected for much longer.

5. Estimation

The main topic discussed under this heading was the estimation of transition proportions, the parameters in the student-flow models. At present these are based on past observations, exactly like input-output coefficients in many economic investigations, and it was agreed that this was the only practical way to get a first approximation to these proportions. It was pointed out, however, that the economic analogy of supply and demand analysis ought to be kept in mind, the proportions based on past data partly reflected the demand of students to move from one branch to another in the educational system and partly the supply of places in those branches. To the extent that supply limitations were important, especially in the higher branches of the system, estimates based on the past would give a distorted picture of student demands if they were in a position to exercise their first preferences; not only would the demand for the more preferred branches be underestimated, but the demand for the less preferred branches might be overestimated since the demand for them comes in part from the exercise of second or third preferences by those who have failed to find a place in their preferred branch.

A number of suggestions were made for handling this problem. In the first place, institutions would usually know how many applications they received as well as how many students they admitted, and might be able to divide the difference between these numbers between the applicants who did not reach their standard and the applicants who had to be rejected owing to the limit on the number of places available. This kind of information would be useful. But it might still be the case that the general knowledge that places were limited discouraged application by weak but suitable candidates. It was suggested that sample surveys of student preferences and attainments might help to get closer to transition proportions base on demand; and the possibility of trying to model the determinants of student demand was also mentioned.

Apart from the question of estimating a set of transition proportions, consideration was also given to the estimation of changes in this set. This problem is familiar to economists in the context of input-output coefficients. Something could be gained by constructing a series of tables of student flows, but these would all be subject to the difficulties just mentioned, due to limitations of supply. Here again it was suggested that it would be necessary to model the determinants of student demand. For example, it might be helpful to regard the wish to reach successively higher rungs on the educational ladder as analogous to a multiple epidemic process in which the change in the transition proportion between any two stages depended partly on the proportion who had made the move in the previous period and so tended to infect others to follow their example, and partly on the proportion who had not made the move in the previous period and so might be susceptible to infection. Again it might be helpful to regard the choice of course at any stage as analogous to a learning process in which the existing distribution over courses is compared with a distribution based on an assessment of their prospects, and students tend continually to move away from the first distribution towards the second at a given speed.

It was generally agreed that educational modelling was still in its infancy and offered immense scope for further research.

6. Aims

The discussion of the aims of educational policy tended to base these aims on the usual, rather limited, considerations: the apparent needs of the productive system; and the apparent preferences of those being educated. It could be argued that there are educational aims over and above these: namely, to bring every individual up to the educational level suited to him and so enable him to enjoy what education can contribute to a happy and adjusted life. This aim, if realised, might bring about large changes in the supply of different economic skills, and so in the prices of these skills, with the result that the apparent demands forecast by the productive system would be wide of the mark if these price changes were not taken into account. This aim might also conflict with individual attitudes, or at least actions, as historically determined. However, it was recognised: (a) that most individuals must earn their living and that the educational system should help them to do this to their satisfaction; and (b) that within any system of inducements that may be offered, individuals should be free to follow the courses that appeal to them. These considerations suggest that the needs of the productive system are important but that the productive system may have to adapt itself to individual attitudes.

From the discussion on educational aims a number of points emerged on which there was general agreement.

First, there are many different educational aims and the weight that should be assigned to each is a problem for the policy maker rather than the model builder. However, the model builder can play an important role in the process of policy formation in so far as he is able to work out problems of feasibility and cost, and can compare the alternative paths by means of which different aims can be realised.

Second, many aims are uncertain, partly because we know relatively little about social and economic dynamics. For example, even if we ignore the possibility of the kind of major adjustment contemplated at the beginning of this section, it is still a difficult matter to estimate future manpower needs, since these depend on future output levels, technology labour mobility, restrictive practices and many other factors. These subjects can be, and are being, studied. In this case the educational model builder must get together with the economic model builder.

Third, aims may be diametrically opposed to one another because of conflicts of interest which must be resolved before a coherent statement of aims can be made. The theory of games was mentioned as a technique which might be useful in this difficult area.

7. Controls

Any system that is to operate satisfactorily must have some form of control to prevent inbalances from building up and to keep it on its intended course. The distinction between automatic controls and controls operated by human agents is not as obvious and clear-cut as it may seem. The thermostat on the one hand and the driver of a motorcar on the other are examples of the two extremes. The price mechanism working under the assumption of universal perfect competition is something between the two, for in this case the economic system is controlled by human agents who have, however, an automatic response to price changes: a rise in price will stimulate supply and curtail demand, and a fall in price will have the opposite effect. In general, the self-interested responses of human agents to price movements are supposed to converge towards an equilibrium solution and so re-establish a balance that has been upset by crop failures or other natural occurrences. Even with this simple model, however, it is not difficult to state conditions under which the human responses to a departure from equilibrium will lead not to convergence but to oscillation: the feed stock cycle is an example of this.

The subject of systematic control has been deeply studied in recent years in connection with engineering control problems. Two of the papers presented to the conference dealt with control from this point of view, not in order to reduce the problems of controlling a programme of educational development to the automatic methods appropriate to controlling, say, a chemical plant, but in order to introduce the general philosophy of the problem of control as it has been developed in control-system engineering. These ideas fit easily into the framework of the discussion given so far, and can be summarised in four propositions: (a) from a knowledge of the operating characteristics of the system and the aims it is intended to serve, suitable control variables must be found; (b) control relationships connecting these variables with the system must be designed in such a way that they act with the speed and intensity, neither too much nor too little, necessary to keep the system close to its intended path; (c) the whole exercise must be based on conditions and knowledge as they are and not on hypothetical conditions and 'perfect' knowledge; (d) the control mechanism should be adaptive, that is to say it should embody a learning process which enables it to adjust itself as new experience is gained. This statement of the problem, which is as relevant to systems controlled by human agents as to automatic-control systems, provides some insight into the fundamental nature of control.

In the discussion, a number of points were made.

First, any human system is likely to be only partially controllable because there may be human responses which are not amenable to any acceptable control variable. This does not, of course, imply that control is unimportant in such systems.

Second, more work is needed on the concept of control variables as applied to educational systems. For example, suppose an improvement in the pay and status of teachers is needed to secure adequate recruitment and retention; how are potential teachers likely to react to specific proposals? Or again, how can administrative arrangements, which themselves form a system, be better adapted to their tasks? And, finally, how far does effective control involve changing the attitudes of students and their parents, so that those parts of the system which are not amenable to controls now become self-regulating?

Third, emphasis was placed on the difficulty of designing control systems. Some of the reasons for this are: (a) the control function which specifies the changes in the control variables in response to the performance of the system must be tractable as well as effective, and it is not easy to satisfy both conditions; (b) the objective function to be maximized or minimized is rarely single-peaked and it is often difficult to find the true maximum or minimum rather than some local peak or trough; (c) controls must operate at the appropriate speed and intensity if they are not to lead to oscillations of ineffectiveness.

8. Conclusion

Those who attended the conference left it with the feeling that the ideas presented held much promise and that the achievements in building quantitative educational models, though modest to date, pointed to important new techniques which would be helpful in the formulation and control of educational programmes. They saw a number of new areas where research is urgently needed, and commended these alike to those engaged in research and to those who support research. They expressed the hope that, even at this early stage, the papers presented in this volume would come to the notice of educators, educational administrators and educational planners, without whose understanding and help educational model building could easily become separated from the very activity it was designed to assist. They thought that at a later stage, when educational model building was a little further developed, there would be a strong case for another kind of conference at which a wider range of interests would be represented.

PART II : SURVEY

A SURVEY OF MATHEMATICAL MODELS IN EDUCATIONAL PLANNING

by Hector Correa

INTRODUCTION

In Figure I the models used for the study of education are classified in two main groups : Micro and Macro Models. The first group is formed by all the models referring to the educational process itself; i.e., to the psychological aspects of learning, to the interaction of teachers and students, and among students. The reader interested in this type of model should consult the references mentioned in Note*(1). Such models will not be considered in the present paper. The reason for this is that micro educational models have not been applied to educational planning and administration.

All models referring to the educational system as a whole, or to parts of it, are included in the group of the macro models. In this

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* The references are presented at the end of this paper.

(1) See references 8, 10, 21, 35.

Figure 1

- Educational models
  - Micro models
  - Macro models
    - Without choice among alternatives
      - Flows of students in the educational system
      - Teachers and class rooms
      - Costing and financing educational plans
      - Educated personnel required for social development
    - With choice among alternatives
      - Optimum enrolment policy
      - Flows of students, teachers buildings and costs
      - Allocation of resources between the economy and the educational system
      - Optimum educational curriculum

case the main elements considered are number of students, of teachers, of buildings, etc. No attention is paid to what happens inside a classroom, or to the psychological processes of the students. The present paper will refer only to this second group of models.

The need to integrate the micro and the macro models is evident. Such integration would advance macro educational models from a stage similar to that of medicine in the middle ages, when medicine was studied without disection, i.e., without studying the interior of the human body. The macro models study education without seeing its organs. On the other hand, the micro models go deeply into the analysis of the organs without seeing the whole body. A mathematical study of education cannot be attempted without considering both types of model.

The mathematical instruments used determine the first sub-classification of the macro educational models. The first sub-group is formed by models that do not consider choice among alternatives. Questions such as the future evolution of the educational system, the consequences of an educational policy, etc., are studied with these models. The second sub-group includes all the models whose main objective is to select an optimum policy, or an optimum path, for the educational system, as a whole, or for parts of it. What has been done is to put educational problems in forms that can be solved with the aid of linear programming techniques.

Macro models without choice between alternatives are divided in four categories dealing with: (a) student flows; (b) teachers and class-rooms; (c) costs and finances; and (d) educated personnel needed for social development.

The forecast of student flows and of the population outside the educational system classified by the level of educational attainment are among the problems considered with the models dealing with student flows. Also, perhaps the most important problem of educational planning can be studied with the aid of these models, i.e., the problem of the adaptation of the educational system to socio-economic development.

The main questions considered in the study of teachers and classrooms are the following: estimation of the number of teachers and class-rooms required to attain the targets of an educational plan, and the problem of an equilibrium path for the number of students in the educational system, determined by demand and supply of teachers.

The two main questions of costs and financing in educational planning are the estimation of the expenditures required to attain the targets of an educational plan, and the estimation of the resources available for financing education. The models available to deal with these questions are presented in the section dealing with costs and financing.

The estimation of the educated personnel needed for social development is the question studied with the models presented in the section dealing with this problem. These models complement the models dealing with student flows, because in the analysis of student flows the problem of adapting the educational system to social needs is studied, while in the present case the problem of evaluating those needs is considered.

In this paper no attempt has been made to present an over-all model integrated with the four sub-models mentioned above. If, for practical reasons, it is desired to do so, no basic modification of the sub-models presented here is needed.

In order to consider the problems of choice of alternatives, it is necessary to integrate several of the aspects dealt with separately before in one over-all model. In the model dealing with enrolment policy, financial resources, population of school age and enrolment are integrated. The model permits the determination of the minimum amount of investment required to open schools for the school age population.

A more general problem is studied with the model dealing with flows of students, teachers, buildings and costs; the problem of optimizing an index of educational products over a period of time, considering as constraints the transition of students from one level to another in the educational system, as well as the stocks of teachers, and the financial resources available.

The general model of choice in the educational system is integrated with a model of economic growth in the section dealing with the allocation of resources between the economy and the educational system. The problem of the objective function is discussed in some detail.

Finally, as an example of some of the possible applications of mathematical techniques to educational planning, the problem of the choice of an optimum curriculum is discussed.

II. MODELS WITHOUT CHOICE AMONG ALTERNATIVES

II-1 Flows of students in the educational system

II-1-1 Introduction

Four questions will be considered in the analysis of the flows of students:

(a) Definition of indices for the comparison of the flows of students at different times and/or places;

(b) Forecast of the evolution of the flows of students;

(c) Forecast of the evolution of the population outside the educational system classified by level of educational attainment; and

(d) Determination of the characteristics that the flows of students must have in order to achieve specified targets of the educational output, including the transition problem (to be defined later).

These four questions have been studied with the aid of two types of models: (1) those that do not explicitly take the educational system into consideration, but rather are based on the relations between population and enrolment; and (2) those that take the educational system into consideration explicitly.

We will first present some basic identities referring to the flows of student. Next the models based on the relations between population and enrolment will be considered; and finally those which take the educational system itself into consideration will be dealt with.

II-1-2 Basic Identities [2]

For the analysis of the flows of students it is useful to consider the following basic identity referring to the students flowing in and out of any sub-division, say a grade, of the educational system:

II-1-2-1

$$n_t + r_t + v_t = g_t + d_t + r_{t+1} + m_t$$

where

n_t is the number of new students in the grade who were in the previous grade during the previous period;

r_t is the number of repeaters; i.e., students in the same grade during the previous period, now taking that grade again;

v_t is the number of students re-entering the educational system, i.e., students who left the educational system several periods before, and re-enter it;

g_t is the number of students successfully finishing the grade; they will be called "graduates";

d_t is the number of drop-outs;

m_t is the number of student deaths;

t is the time of reference.

It should be observed that several time dimensions are considered in identity 2-1* and that all of them are denoted by t. For example, the t in g_t refers to a period of study one later than that in n_t or r_t, while t in d_t and m_t refers to the whole period of study, because these two variables represent the total number of events occurring at any time during the period of study.

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[2] See references 14-18.

* Reference to a formula inside a section will be made with the last two figures only; for example, formula II-1-2-1, will be referred to as formula 2-1 in section II-1.

It is useful to refer the variables in identity 2-1 to the number of students at some instant between the beginning and the end of period t. In this case, the events are classified in two groups: those happening before the instant chosen, and those happening after. Despite the risk of confusion, we will again denote by t an instant between the beginning and the end of period t. In this case, identity 2-1 takes the following form:

II-1-2-2 $$s_t = n_t + r_t + v_t^1 - d_t^1 - m_t^1$$

II-1-2-3 $$s_t = g_t + r_{t+1} + d_t^2 - v_t^2$$

where	s_t	is the number of students at instant t;
	d_t^1	is the number of students dropping out between the initiation of the period and instant t; i.e. drop-outs before t;
	d_t^2	is the number of drop-outs between instant t and the end of the period, i.e., drop-outs after t;
	v_t^j	is the number of students re-entering the educational system; j = 1 before instant t j = 2 after instant t
	m_t^j	is the number of deaths; j = 1 before instant t j = 2 after instant t.

When s_t is eliminated from identities 2-2 and 2-3, the result is again identity 2-1.

The main reason for introducing identities 2-2 and 2-3 is that they can be used as a frame of reference to aggregate several subdivisions of the educational system, for example, several grades.

Identity 2-1 cannot be used for this purpose.

In order to aggregate several grades of the educational system the notion of structure is used. This is best introduced with an example. In tables 1 and 2 the basic data for the aggregation of three consecutive grades is presented. The arrays of numbers in these tables form a structure centered around year 0. This control year will be called the pivotal year. For this reason the figures in the tables will be called structure 0. In tables 1 and 2, it can be seen that the previous structure would be centered around pivotal year -3, and the next around pivotal year 3. In general, if k grades are aggregated, the structure would be centered around pivotal years ..... t-2k;t-1k; t; t+k..........

Table 1

Data for the aggregation of three consecutive grades

(structure 0: total, new in grade, repeaters)

Grade		-2	-1	0	1	2	3
1	Total.........	100	105	100			
	New...........	90	96	92			
	Repeaters.....	10	9	8	8		
2	Total.........	89	89	93	98		
	New...........		81	85	91		
	Repeaters.....		8	8	7	8	
3	Total.........		79	79	81	86	
	New...........			73	76	80	
	Repeaters.....			6	5	6	7

Table 2

Data for the aggregation of three consecutive grades

(structure 0: total leavers, drop-outs and deaths)

Grade		-2	-1	0	1	2
1	Total	10	12	11		
	Drop-outs	8	10	11		
	Deaths	2	2	0		
2	Total		8	10	10	
	Drop-outs		7	8	9	
	Deaths		1	2	1	
3	Total			74	75	79
	Graduates			72	72	77
	Deaths			2	3	2
	Drop-outs	8	17	19	9	
	Graduates			72	72	77
	Deaths	2	3	4	4	2

In structure 0 the history of the total number of students enrolled in year 0 is given. This total number is

$$s_o = 282.$$

Table 1 presents the history of those who remained in the system, while table 2 of those who left the system. The inputs in structure 0 are given in table 1. They are the new students entering first grade in years -2, -1 and 0., i.e.,

$$n_o = 90 + 96 + 102 = 288;$$

plus the repeaters in first grade year -2, second grade year -1 and third grade year 0, i.e.,

$$r_o = 10 + 8 + 6 = 24.$$

The difference $n_o + r_o - s_o$ gives the number of students who left structure 0. This can be verified in table 2, which gives the total number of leavers, grade by grade and year by year. For example, the total number of school leavers from first grade in year -1 is

$$12 = 105 - 8 - 85.$$

The number 12 appears in Table 2, while the numbers to the right of the equal sign appear in Table 1.

It is now clear that, for structure 0, the variables in formulas 2-2 and 2-3 take the following values from Table 1:

$$s_o = 110 + 93 + 79 = 282$$

$$n_o = 90 + 96 + 102 = 288$$

$$r_o = 10 + 8 + 6 = 24$$

$$r_3 = 8 + 8 + 7 = 23; \text{ and}$$

$$v_o^1 = 0;$$

from Table 2:

$$d_o^1 = 8 + 17 = 25$$

$$m_o^1 = 2 + 3 = 5$$

$$g_o = 72 + 72 + 77 = 221$$

$$d_o^2 = 19 + 9 = 28$$

$$m_o^2 = 4 + 4 + 2 = 10$$

$$v_o^2 = 0.$$

The above identities refer to one process in the educational system, one level say. The educational system is composed of a set of interdependent processes. We will take it that there are 1,2.....,I interdependent processes . The first 1,2,.... A processes are considered the most elementary ones, both persons with and persons without previous education can enter in any of these processes. Only those with previous education can enter processes $i = A + 1,.....,I$. It should be observed that the enumeration above does not mean that students in process i were in process i - 1 in the previous period. Students in process i might come from any other process $i = 1,2....,I$. For example, if the last grade of elementary school is $i = 6$, the first year of general high school could be $i = 7$, and the first of vocational high school $i = 13$. Thus students from $i = 6$ in any year could be in processes $i = 6$, 7 or 13 in the next year.

When the variables in the identities 2-2 and 2-3 refer to process i, a superscript, i, will be used.

Using identities 2-2 and 2-3 as a starting point it is possible to give a precise definition of the output of the educational system. In this definition it should be observed that there are several ways in which a person can have an educational level. The simplest one is if the person graduates from that level and leaves the educational system. In addition, in some cases it is said that a person who drops out from educational process i has educational level i. People who drop out from levels immediately above i are considered to have educational level i; for example, high school drop-outs are considered to have only elementary education. These three elements must be considered in the definition of the output of the educational system.

The following notation will be used:

o^{i}_{t+k} is the number of people with educational level i, leaving the system between t and t+k;

g^{-1i}_{t} is the number of graduates from level i who leave the system between t and t + k, without further studies;

d^{2ij}_{t} is the number of drop outs after pivotal year t, process j, who are considered part of the output of educational level i;

d_{t+k}^{1ij} is the number of drop outs before pivotal year t+k, process j, who are considered part of the output of educational level i.

With the above notation we have

II-1-2-4 $$o_{t+k}^{i} = g_t^{1i} + \sum_{j=1}^{I} d_{t+k}^{2ij} + \sum_{j=1}^{I} d_t^{1ij}$$

Several indices can be defined, taking identities 2-2 and 2-3 and definition 2-4 as a starting point for the analysis of the flows of students. No details will be given here [3].

II-1-3 Population and Enrolment

The simplest model for the analysis of the flows of students reduces to the following equation:

II-1-3-1 $$s_t = f(P_t)$$

that is, the number of students at time t, S_t, is some function of the population at that time, P_t.

Aggregate values for s and P are not considered.

Disaggregation can proceed along one or several of the following lines:

(a) Age;

(b) Sex;

(c) Grade;

(d) Type of education (general; vocational; in law, economics,etc.);

(e) Type of students (repeaters, non-repeaters, etc.);

(f) Geographical and social divisions (urban and rural; family's social class, etc.).

When disaggregation along one of the lines previously mentioned is used the equations in 3-1 have to be modified. For example, if disaggregation by age and sex is used, equations of the form

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[3] See references 14, 18, and 45.

II-1-3-2 $$s_{thi} = f_{thi}(P_{thi})$$

$h=1,......,H$ denotes age

$i=1,2$ denotes sex

are used.

Linear functions with zero intersection have often been used to relate the number of students in the educational system to population. in this case, the function in (2) takes the form

II-1-3-3 $$s_{thi} = \tau_{thi} P_{thi}$$

where τ are quantities that might or might not be constant, but whose variation is independent of P.

If equations 3-1, 3-2 or 3-3 are compared with identities 2-1, or 2-2 and 2-3, it will be observed that several variables referring to the flows of students in the educational system have been omitted. On the other hand, it is clear that the model in this section can be extended to include the omitted variables. For example, equations referring to students leaving the educational system as drop outs or as graduates could be established. It is not, however, possible to do so without considering the identities in section 2 implicitly or explicitly. For this reason we will not explore this matter here.

In equations 3-1, 3-2 or 3-3 the model can be used to compare the educational conditions between times and/or places. The coefficients τ can be used as indices for this purpose. These indices complement the information obtained with the indices mentioned in section 2-2, because those indices refer only to the internal process of the educational system, while indices such as τ establish a relation between students and total population.

In addition, if a complete set of equations (1) or (2) are known, as well as the projections of the population (classified by sex and age if required), it is possible to prepare complete projections of the number of students in the educational system.[4]

Three main limitations of the above model should be mentioned. First, when used for comparison of the educational conditions at several different places and/or times, it might lead to false conclusions because it reflects not only educational differences, but also differences in the population structure. Second and more important, it does not give any insight in the process in the educational system itself. Finally, the model does not give any idea of possible modifications of the proportions T, nor of the factors causing them.

The model in equation 3-3 can be slightly modified to give tables of school life for the total populations.[5] The basic equation in this case has the following form:

II-1-3-4 $$\hat{s}_{th} = T_{th} \hat{P}_{th}$$

$\hat{s}_{th}$ is the "stationary school population" of age h in year t;

T_{th} is the enrolment rate of people of age h in year t;

$\hat{P}_{th}$ is the number of living people age h to h+1 in the stationary population in year t. The expression stationary population should be taken in the demographic sense. (P_{th} corresponds to column L_x in the life tables)

Comparing equations 3-3 and 3-4, we observe that the usual estimates of s_{th} are obtained when the usual values of P_{th} are used, while estimates of the stationary school population are obtained if values of $\hat{P}_{th}$ are used.

When the values corresponding to the stationary school population are known, it is possible to construct "Tables of the second life for the Enrolled Population".[5] They represent the enrolled population classified by age columns similar to those appearing in any life table. Among these columns perhaps the most interesting are those giving the average number of school years remaining to people age exactly h who are alive and enrolled in school, and the probability that a person enrolled during a particular age interval will drop out of school before the next age interval is reached.

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[4] See reference 37.

[5] " " 38.

The tables of school life for the total and for the enrolled population have the same uses, advantages and most of the disadvantages of the model in equation 3-3. The only advantage of the table over this model is that they eliminate the influence of population changes on educational conditions.

II-1-4 Models which explicitly consider the educational system

II-1-4-1 - Comparison of the assumptions of different models.

The analysis to be made in this section 4-1 is based upon identities 2-2 and 2-3 written in the following form;

II-1-2-2 $$s_t = n_t + r_t + v^1_t$$

II-1-2-3 $$s_t = g_t + d_t + r_{t+1} + m_t - v^2_t$$

In this form of identities 2-2 and 2-3, the instant of reference t is the beginning of the educational period.

In the simplest models[6] the following form of identities 2-2 and 2-3 is used:

$$s_t = n_t$$

$$s_t = (g_t - g^1_t) + (g^1_t + d_t)$$

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[6] See references 10, 12, 19, 37.

i.e., it is assumed that

$$r_t = m_t = v_t^i = o \qquad i = 1,2;$$

and that students in one grade can continue their studies or drop out of the system. No distinction is made between drop outs and graduates.

The above equations have been extended to take account of repeaters,[7] and deaths.[8]

In either of the two cases mentioned above it is assumed that

$$v_t^i = o \qquad i = 1,2;$$

and no distinction is made between graduate and drop out leavers.

The aggregate form of equations 2-2 and 2-3 has been used[9] with the assumption that

$$v_t^i = o, \quad i = 1,2.$$

II-1-4-2 - The basic model[10]

Starting with identities 2-2 and 2-3, definition 2-4 and the interrelations between different educational levels, several models of the educational system can be constructed. In its general form the simplest model can be written in the following way:

II-1-4-1 $$s_t^i = n_t^i + r_t^i - d_t^{1i} - m_t^{1i}$$

II-1-4-2 $$s_t^i = g_t^i + d_t^{2i} + r_{t+k}^i + m_t^{2i}$$

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[7] See references 2, 25, 30, 45.

[8] " " 39.

[9] " " 14 - 18 .

[10] In this section a generalization of the analysis made in reference 14 is presented.

II-1-4-3 $$n_t^i = \sum_{j=1}^{I} f(g_{t-k}^j) + f(P_{t,1};\ P_{t,2};\ \ldots;\ P_{t,H}) \qquad i = 1,\ldots\ldots,A$$

II-1-4-4 $$n_t^i = \sum_{j=1}^{I} f(g_{t-k}^j) \quad i = A+1,\ldots\ldots\ldots,I$$

II-1-4-5 $$r_t^i = f(s_{t-k}^i)$$

II-1-4-6 $$d_t^{1i} = f(n_t^i;\ r_t^i)$$

II-1-4-7 $$d_t^{2i} = f(s_t^i)$$

II-1-4-8 $$m_t^{1i} = f(n_t^r;\ r_t^i)$$

II-1-4-9 $$m_t^{2i} = f(s_t^i)$$

Where P_{th} is the population aged h=1,......,H in year t;
and f( ) is some function of ( ). Each time that this symbol is used in equations 4-1 to 4-9, it refers to a different function.
The other symbols have already been explained.

The only exogenous variable in the model in equations 4-1 to 4-9, are the P_{th} for all the age groups, h=1,......H, in which people may enter the educational system.

Given the future values of P_{th} and the forms of the functions in equations 4-1 to 4-9, the model permits us to forecast the future evolution of student flows. In this sense, the first of the problems stated in section 1 has been solved.

If definition 2-4 is added to this model the future evolution of the output of the educational system may be studied.

We will next study the forecast of the educational structure of the population classified by level of education.

Let G_t^i be the number of non-students in the population of educational level i. The future evolution of G_t^i is given by the following equations :

II-1-4-10 $$G^i_{t+k} = (1 - \mu^i_G)G^i_t + (1 - \mu^i_o)o^i_{t+k}$$

where μ^i_G is the death rate of those in the population with educational level i

μ^i_o is the death rate of the output of educational level i between t and t+k after they have left school.

The introduction of the survival factor, $(1 - \mu^i_o)$, follows from the definition of o^i_{t+k}. According to this definition, all school leavers between t and t+k are included in the output of the educational system, o^i_{t+k}, from the moment that they leave school. Some of them die before t+k at which time, the stock, G^i_{t+k}, is evaluated. The rate of these deaths is μ^i_o.

Equations 4-1 to 4-10 and definition 2-4 (of the output of education), permit us to forecast the evalution of the educational structure of the population.

A different model is required to determine the characteristics which the flows of students must have in order to achieve the specified targets for the educational output. The equations of this model are 4-1, 4-2, 4-5, 4-8 and 4-9 presented before and also :

II-1-4-11 $$o^i_{t+k} = g^{1i}_t + d^{2i}_t + d^{1i}_{t+k}$$

II-1-4-12 $$g^{1i}_t = f(o^i_{t+k})$$

II-1-4-13 $$d^{2i}_t = f(o^i_{t+k})$$

II-1-4-14 $$g^i_t = g^{1i}_t + \sum_j n^{j*}_{t+k}$$

where the values of j^* are those levels to which it is possible to transfer from level i.

With this model, given the target value of o_t^i, it is possible to determine the values that the different variables in the system, s_t^i for instance, must have in order to achieve such targets. The values of the variables obtained in this way need not agree with their present values nor with the likely evolution of their values in the future. This creates the problem of the transition[11] from the present conditions of the educational system, and its likely evolution in the future, to the conditions required to attain the targets for the output of the system.

In mathematical terms this means that we have a model formed by equations 4-1 to 4-9 and 4-11 to 4-14. Such a model has 9 x 1 variables and 12 x I equations. To solve this system 3 x 1 additional variables must be introduced. 3 x I parameters of the functions in the model would usually be considered as variables. The transition problem is thus solved.

The above models solve, in theory, the problems stated in Section I. There are however, two questions deserving further attention: first the relation between population and new entrants; and second the analysis of the special case of the models introduced here when the functions are assumed to be linear.

II-1-3 Population and new entrants[12]

In this section we are concerned only with the new entrants in grades i = 1, ... A, and not with those students coming to these grades from some other process in the educational system. Let equation 4-3 be written

$$n_t^i = \sum_{j=1}^{I} f(g_{t-k}^j) + \bar{n}_t^i \qquad i = 1,\dots\dots,A.$$

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[11] See references 10, 14, 19, 41.

[12] " " 14.

With $\bar{n}^i_t = f(P_{t,1}, \ldots\ldots P_{t,H}) \quad i = 1,\ldots\ldots,A.$

i.e. $\bar{n}^i_t$ is the number of new entrants in the educational system (Note that n^i_t the number of new entrants in a structure i, as defined in section 2).

The number of new entrants can be disaggregated by age as follows

$$\bar{n}^i_t = \sum_{h=1}^{H} \bar{n}^i_{t,h} \qquad i = 1,\ldots\ldots,A.$$

where $\bar{n}^i_{t,h}$ is the number of people aged h entering the educational system for the first time. In addition, assume that

$$\bar{n}^i_{t,h} = T^i_{t,h} P_{t,h}$$

(i.e., the number of new entrants in the educational system is a constant proportion of the population of the appropriate ages)

and assume that $P_{t+1,\ h+1} = P_{t,h}(1-\mu)$

where μ is the annual mortality rate of the population.

In this section we want to emphasize the interdependence of the different values of $T^i_{t,h}$ for different values of t and h. In fact, they are subject to the following condition:

II-1-4-15 $$\sum_{i=1}^{A} \sum_{j=1}^{J} T^i_{t+j;\ h+j} \leq 1 \quad \text{for any } j > 0.$$

To prove 4-15, observe that

II-1-4-16 $$\sum_{i=1}^{A} \sum_{j=i}^{J} n^i_{t+j;\ h+j} (1-\mu)^{J-j} \leq P_{t+J;\ h+J}.$$

Relationship 4-16 is self evident; it says that the number of survivors among new entrants belonging to the generation aged h+J in year t+J must be less than or equal, the number of persons in that generation $P_{t+J,\ h+J}$.

From the definition of $T^i_{t,h}$ we have

$$\sum_{i=1}^{A} \sum_{j=1}^{J} T^i_{t+j;h+j}\ P_{t+j;h+j}\ (1-\mu)^{J-j} \leqslant P_{t+J;\ h+J}$$

and since

$$P_{t+j;\ h+j}\ (1-\mu)^{J-j} = P_{t+j;\ h+J} \qquad j=1,\ldots\ldots,J$$

equation 4-15 follows.

The interdependence of the new entrant ratios must be considered when their future values are estimated.

II-1-4-4 The linear model

The model in equations 4-1 to 4-9 written with linear functions takes the following form[13]

II-1-4-17 $$s^i_t = n^i_t + r^i_t - d^{1i}_t - m^{1i}_t$$

II-1-4-18 $$s^i_t = g^i_t + d^{2i}_t + r^i_{t+k} + m^{2i}_t$$

II-1-4-19 $$n^i_t = \sum_{\substack{j=1\\ i\neq j}}^{I} \gamma^{ij}\ g^j_{t-k} + \bar{n}^i_t \qquad i = 1,\ldots\ldots\ldots,A$$

II-1-4-20 $$n^i_t = \sum_{\substack{j=1\\ i\neq j}}^{I} \gamma^{ij}\ g^j_{t-k} \qquad i = A+1,\ldots\ldots,A.$$

---

[13] See reference 14.

II-1-4-21 $$r^i_{t+k} = \gamma^{ii} s^i_t$$

II-1-4-22 $$d^{1i}_t = \delta^{1i} (n^i_t + r^i_t)$$

II-1-4-23 $$d^{2i}_t = \delta^{2i} s^i_t$$

II-1-4-24 $$m^{1i}_t = \mu^{1i} (n^i_t + r^i_t)$$

II-1-4-25 $$m^{2i}_t = \mu^{2i} s^i_t$$

where γ^{ij} is the proportion of graduates from level j going to level i for $i \neq j$, and the proportion of repeaters in level i for $i=j$;

δ^{ij} is the proportion of drop-outs from level j;

$i = 1$ before the pivotal year
$i = 2$ after the pivotal year

μ^{ij} is the mortality rate of students in level j;

$i = 1$ before the pivotal year
$i = 2$ after the pivotal year.

The meanings of the other symbols are as above.

The model just presented has 8 I equations, and 8 I + A, variables, A of them being exogeneous.

Future evolution of the number of students enrolled[14]

To simplify the presentation some changes in the notation presented above will be introduced.

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[14] See references 2, 25, 30, 31.

II-1-4-26 $$g_t^i = \alpha^i \, s_t^i$$

with $$\alpha^i = (1 - \delta^{2i} - \gamma^{ii} - \mu^{2i}) \text{ ; and}$$

II-1-4-27 $$s_t^i = \beta^i \, (n_t^i + r_t^i)$$

with $$\beta^i = (1 - \delta^{1i} - \mu^{1i})$$

In equation 4-27, replace n_t^i by its value in terms of s_{t-k}^j, $j = 1, \ldots, I$, from equations 4-20 and 4-26; and replace r_t^i by its value in terms of s_{t-k}^i from equation 4-21; giving

$$s_t^i = \beta^i \sum_{\substack{j=1 \\ j \neq i}}^{I} \gamma^{ij} \, \alpha^j \, s_{t-k}^j + \beta^i \, \alpha^{ii} \, s_{t-k}^i$$

$$i = A+1, \ldots, I.$$

A similar equation, but including $\bar{n}_t^i$ can be written for $i=1,\ldots,A$. From this equation we can write the following matrix equation :

II-1-4-28 $$\left[s_t \right] = \left[\Omega^{11} \right] \left[s_{t-k} \right] + \left[\bar{n}_t \right]$$

where $\left[s_t \right]$ is a vector with I components s_t^i

$\left[\Omega^{11} \right]$ is a I x I matrix with element:

$$\omega^{ij} = \begin{cases} \beta^i \, \gamma^{ij} \, \alpha^j & \text{for } i \neq j \\ \beta^i \, \gamma^{ii} & \text{for } i = j \end{cases}$$

$\bar{n}_t$ is a vector with I components $\bar{n}_t^i$, only the first A of which are non-zero.

From the system of finite difference equations in 4-28, the variables $\left[s_t \right]$ can be expressed as functions of $\bar{n}_t$. Thus the problem of forecasting the number of students enrolled has at least in theory been solved.

Future evolution of the output of the educational system

The output of level i was defined, in equation 2-4, as

$$o^{i}_{t+k} = g^{1i}_{t} + \sum_{j=1}^{I} d^{2ij}_{t} + \sum_{j=1}^{I} d^{1ij}_{t+k}$$

In this section, each of the elements of equation 2-4 will be related to s^j_t for j=1, ...,I. To begin with, from the meaning of the coeficients γ^{ij} in equation 4-19 it may be seen that the number of students graduating from a level of the educational system and remaining in the system as new entrants in some other level is

$$\gamma^{ij} g^{j}_{t} = \sum_{\substack{i=1 \\ i\neq j}}^{I} \gamma^{ij} g^{j}_{t}$$

i.e., the number of graduates from level j leaving the educational system and becoming part of the output is

$$g^{1j}_{t} = (1- \gamma^{1j}) g^{j}_{t} s^{j}_{t}$$

i.e. from equation 4-26

II-1-4-29 $$g^{ij}_{t} = (1 - \gamma^{1j}) \alpha^{j} s^{j}_{t} .$$

Next, let us define

$$d^{1ij}_{t} = \delta^{1ij} (n^{j}_{t} + r^{j}_{t})$$

where δ^{1ij} is the proportion of the inputs in level j who drop out and are considered part of the output of level i.

Using equation 4-27 we get

II-1-4-30 $$d_t^{1ij} = \frac{\delta^{1ij}}{\beta^j} s_t^j.$$

Finally, let us define

II-1-4-31 $$d_t^{2ij} = \delta^{2ij} s_t^j$$

where δ^{2ij} is the proportion of students in level j who drop out and are considered part of the putput of level j.

If equations 4-29, 4-30 and 4-31 are used in equation II-1-2-4, we have

$$o_{t+k}^i = (1 - \gamma^i) \alpha^i s_t^i$$
$$\sum_{j=1}^{I} \delta^{2ij} s_t^j + \sum_{j=1}^{I} \frac{\delta^{1ij}}{\beta^j} s_{t+k}^j$$

i.e.,

II-1-4-32 $$\left[o_{t+k}\right] = \left[(1-\gamma^i) \alpha^i\right]\left[s_t\right] + \left[\delta^2\right]\left[s_t\right] + \left[\frac{\delta^1}{\beta}\right]\left[s_{t+k}\right]$$

where the terms in the equation have obvious matrix interpretations.

Finally, we can use equation 4-28 in equation 4-32 above, and obtain

II-1-4-33 $$\left[o_{t+k}\right] = \left[\bar{\Omega}^{21}\right]\left[s_t\right] + \left[\frac{\bar{\delta}^1}{\beta}\right]\left[\bar{n}_{t+k}\right]$$

where $\bar{\Omega}$ 21 is obtained from the matrices on the right hand side of 4-32 and the matrices in 4-28.

Since it is possible to express s_t^i as a function of $\bar{n}_t^i$, equation 4-33 expresses o_t^i as a function of $\bar{n}_t^i$. The problem of the evolution of the output of the educational system is thus solved.

Future evolution of the educational structure of the population[15]

The evolution of the number of persons outside the educational system and with educational level i given in equation 4-10 is the following

$$G_{t+k}^i = (1-\mu_G^i)\, G_t^i + (1-\mu_o^i)\, o_{t+k}^i.$$

Using this equation, and equations 4-28 and 4-33 we can write

II-1-4-34 $$\begin{bmatrix} s_t \\ G_t \end{bmatrix} = \begin{bmatrix} \Omega^{11} & \Omega^{12} \\ \Omega^{21} & \Omega^{22} \end{bmatrix} \begin{bmatrix} s_{t-1} \\ G_{t-1} \end{bmatrix} + \left[\begin{array}{c} \delta^1 \\ \hline \beta \end{array}\right] \begin{bmatrix} \bar{n}_t \\ 0 \end{bmatrix}$$

where

$\begin{bmatrix} s_t \\ G_t \end{bmatrix}$ is a vector with 21 compenents, I of which are s_t^i and G_t^i respectively;

$\begin{bmatrix} \bar{n}_t \\ o \end{bmatrix}$ is a vector with 21 components, the first I of which are $\bar{n}_t^i$ and the other zero;

Ω^{11} is an I x I matrix defined in equation 4-28;

Ω^{12} is an I x I matrix of zeros;

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[15] See references 2, 1[illegible], 30, 31, 40.

$$\Omega^{21} = \left[1 - \mu_o^i\right] \left[\bar{\Omega}^{21}\right]$$

where $\left[1 - \mu_o^i\right]$ is an I x I matrix with $i-\mu_o^i$ the main diagonal and zeros elsewhere;

Ω^{21} is the I x I matrix defined in equation 4-33; and

$\begin{bmatrix} 1 \\ \\ \frac{\delta^1}{\beta} \end{bmatrix}$ is a 2 I x I matrix. The first I rows and I columns form a I x I matrix with ones in the main diagonal and zeros elsewhere. The last I rows and I columns are equal to

$$\left[\frac{\delta^1}{\beta}\right] = \left[1 - \mu_o^i\right]\left[\frac{\delta^1}{\beta}\right]$$

where the factor matrices on the right hand side have been previously defined.

The matrix Ω, formed by the matrices Ω^{ij} in equation 4-34, can be used as the starting point to form a Markov chain for the analysis of the educational system and its output. For this, the elements of matrices Ω^{ij} must be interpreted as probabilities. In addition, the probability of dying must be explicitly included, and some absorbing states representing the dead must be included. With this procedure the results obtained for absorbing Markov chains may be used to study the educational system and its output.

Characteristics which the flows of students must have in order to achieve specified targets for the educational output[16]

In order to study this problem with the linear model, some additional simplifications will be used: first, only the disaggregated case will be considered; second, it will be assumed that the educational system is formed by only one sequence of processes, i.e., the graduates from process i go to process i + 1 or leave the system; finally, it will be assumed that the drop-outs from level i form part of the output of level i.

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[16] See reference 39.

With this simplification the model reduces to the following equations:

II-1-4-35 $$o^i_{t+1} = (1 - \gamma^i) \ g^i_t + d^i_t$$

II-1-4-36 $$s^i_t = n^i_t + r^i_t$$

II-1-4-37 $$s^i_t = g^i_t + d^i_t + r^i_{t+1} + m^i_t$$

II-1-4-38 $$n^i_t = \gamma^{i-1} g^{i-1}_{t-1}$$

II-1-4-39 $$r^i_{t+1} = \gamma^{ii} \ s^i_t$$

II-1-4-40 $$m^i_t = \mu^i \ s^i_t$$

Besides the simplifications introduced there is an important difference between the model above and the model in equations 4-1, 4-2, 4-5, 4-8, 4-9 and 4-11 to 4-14. In equation 4-12, the number of graduates is a factor of the output required from the educational system. No such direct relationship exists in the model in equations 4-35 to 4-40. Instead, a direct relationship between n^i_t and g^{i-1}_{t-1} is introduced in equation 4-38. This permits us to obtain an interesting solution for the system of equations.

Using equations 4-25, 4-37, 4-39 and 4-40 we obtain the following matrix equation :

II-1-4-41 $$\left[r \right] \left[s_t \right] = \left[\gamma^i \right] \left[g_t \right] + \left[o_{t+1} \right]$$

where $\begin{bmatrix}\Gamma\end{bmatrix}$ is an I x I matrix with $(1-\delta^i - \gamma^{ii})$ main diagonal and zeros elsewhere.

$\begin{bmatrix}\gamma^i\end{bmatrix}$ is an I x I matrix with γ^i in the main diagonal and zeros elsewhere; and the other elements in equation 4-41 are vectors with componants s_t^i, g_t^i, and o_t^i respectively.

From equations 4-36, 4-38 and 4-39 we can write

II-1-4-42 $$\begin{bmatrix}\Lambda\end{bmatrix}\begin{bmatrix}s_t\end{bmatrix} = \begin{bmatrix}\gamma^i\end{bmatrix}\begin{bmatrix}g_{t-1}\end{bmatrix} + \begin{bmatrix}\Lambda\end{bmatrix}\begin{bmatrix}\gamma^{ii}\end{bmatrix}\begin{bmatrix}s_{t-1}\end{bmatrix}$$

where $\begin{bmatrix}\Lambda\end{bmatrix}$ is an I x I matrix with 1 in the diagonal above the main one and zeros elsewhere; and

$\begin{bmatrix}\gamma^{ii}\end{bmatrix}$ is an I x I matrix with γ^{ii} in the main diagonal and zeros elsewhere.

From 4-41 and 4-42`

$$\begin{bmatrix}\Gamma\end{bmatrix}\begin{bmatrix}s_t\end{bmatrix} = \begin{bmatrix}\Lambda\end{bmatrix}\begin{bmatrix}s_{t+1}\end{bmatrix} - \begin{bmatrix}\Lambda\end{bmatrix}\begin{bmatrix}\gamma^{ii}\end{bmatrix}\begin{bmatrix}s_t\end{bmatrix} + \begin{bmatrix}o_{t+1}\end{bmatrix}$$

II-1-4-43 $$\begin{bmatrix}s_t\end{bmatrix} = \begin{bmatrix}K\end{bmatrix}\begin{bmatrix}\Lambda\end{bmatrix}\begin{bmatrix}s_{t+1}\end{bmatrix} + \begin{bmatrix}K\end{bmatrix}\begin{bmatrix}o_{t+1}\end{bmatrix}$$

with $$\begin{bmatrix}K\end{bmatrix} = \left\{\begin{bmatrix}\Gamma\end{bmatrix} + \begin{bmatrix}\Lambda\end{bmatrix}\begin{bmatrix}\gamma^{ii}\end{bmatrix}\right\}^{-1}.$$

Using the recurrence system in 4-43 it is possible to show that

II-1-4-44 $$\left[s_t\right] = \sum_{i=0}^{I=1} \left(\left[K\right]\left[\Lambda\right]\right)^i \left[K\right]\left[o_{t+1+i}\right].$$

Equation 4-44 permits us to obtain the number of students in the educational system s_t when I - 1 vectors of future output of the system are known.

The transition problem will not be studied in the case of the linear model.

II-1-4-5 Extensions of the basic model

The main reasons for criticizing the model in section 4-2 are that it considers the educational process isolated from other social processes, except population growth; and that it disregards the influence of supply factors on the educational process. The socio-economic factors influencing the population to enter and remain in school are completely omitted. The same is true of factors that can be included in the supply of education such as teachers, buildings, etc.

Two types of extension of the basic model have been proposed in order to avoid such criticisms. In the first,[17] it is assumed that the parameters in the equations of the model change according to some type of stochastic process; for example, the following process is proposed:

$$\Delta x = ax \ (1-b-x)$$

where x could be the proportion of people now entering schools T, or the proportion passing from one level to a higher one, ω^{ij} $(i \neq j)$, and a and b are constants.

This procedure is attractive, but we should not forget that the use of a stochastic method implies a lack of knowledge of the factors influencing the process under study. Actually, the situation is not as bad as this, and it is possible to estimate different values for a and b, for example, for different social classes.

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[17] See reference 39.

Another approach[18] attempts to directly introduce per capita income as one of the factors explaining the changes in the parameters of the functions of the basic model. Lack of data and of statistical and econometric methodology are the most important obstacles in this case. It is possible that aggregated models will be more appropriate for this type of analysis.

Further extensions of the model would involve the internal workings of education; in which case the results of learning theory, theory of the interaction between teachers and students, etc., must be integrated.

The results presented in section II-1-4-4 can be modified to include the possibility of changes in the parameters of the linear functions in the model in equations 4-17 to 4-25. In this case, the product of several different matrices appears in the final formulae instead of a constant matrix raised to a power[19].

In the next sections supply factors, such as teachers and buildings will be considered.

II-2 Teachers and class-rooms

II-2-1 Introduction

Three questions will be considered in this study of teachers and class-rooms. First, a basic identity relating these two elements of the educational system with students and the curriculum will be introduced. This identity will permit us to pass to the second question: the estimation of the number of teachers and class-rooms required to attain the targets of an educational plan. Finally, a model for the study of the equilibrium path between supply and demand of teachers will be considered.

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[18] See reference 14. For statistical analysis of the factors influencing the parameters of the educational model see references 10, 24, 34, 36.

[19] For instance, see reference 39.

II-2-2 Basic identity[20]

To simplify the presentation we will assume, at least at the beginning, that the educational system is still what it was up to some years ago: a process of interaction between one teacher and several students, in a class-room and for a fixed period of time.

Consider the students in class i meeting daily for several periods. For this class the following identity holds

$$s^i = \frac{N^i}{M^i}$$

where s^i is the number of students in the class, i.e., the size of the class;

N^i is the sum of the number of periods attended by each student in the class

M^i is the number of periods for class i in the school curriculum.

If several classes are considered, the average size of class is

$$\bar{S} = \frac{\sum_i s^i M^i}{\sum_i M^i}$$

The interest of this definition is that it can be related to teachers and class-rooms. It will be observed that

$$s = \sum_i s^i$$

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[20] See reference 14-18.

and that the average number of periods attended by each student is

$$p_s = \frac{\sum_i s^i M^i}{\sum_i s^i}$$

M^i; the number of periods that class i receive, is equal to the number of periods taught by the teachers of class i; i.e.,

$$M^i = T^i \; p_c^i$$

where T^i is the number of teachers in class i;

p_c^i is the average number of periods taught by the teachers of class i.

Furthermore, if $T = \sum_i T^i$

and if $\sum M^i = T \; p_c$

then $\bar{S} = \dfrac{s \; p_s}{T \; p_c}$

i.e., the average size of a class is a weighted student - teacher ratio.

In addition, if we study M^i in terms of the number of class-rooms, and in terms of the number of periods during which a class-room is used, we obtain the following identity

II-2-2-1 $$\bar{S} = \frac{s \; p_s}{T \; p_c} = \frac{s \; p_s}{R \; p_r}$$

where R is the number of class-rooms; and

p_r is the average number of periods during which class-room is used.

Several modifications of the previous formulae may be used in order to take into consideration the different subjects in the curriculum, and the use of teaching aids, such as laboratories, closed circuit T.V., teaching machines, etc.

II-2-3 Estimation of the number of teachers and class-rooms required to attain the targets of an educational plan [21]

If, in an educational plan, target values for the number of students are given, the number of teachers and class-rooms can be estimated with the following formulae: *

$$T = [\rho] \quad [s]$$

and

$$R = [\bar{\rho}] \quad [s]$$

where $[\rho]$ is an I x I matrix with elements

$$\rho^i = \frac{p_s^i}{\bar{s}^i \; p_d^i}$$

for i = 1, ..., I, the different levels of the educational system;

$[\bar{\rho}]$ can be defined in a similar way; and

$[s]$ is an I dimensional vector with components s^i, the target values for the number of students in level i.

II-2-2-4 Supply and demand of teachers and the growth of the educational system [22]

Study of the supply and demand of teachers and growth of the

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* T and R are vector quantities

[21] See references 14 - 39.

[22] " " 4 - 9.

educational system is interesting from a theoretical point of view. A superficial analysis immediately shows clear similarities between this question and the economic problems of savings and the growth of income.

The problem is that the number of teachers graduating in the educational system determines the increments of the number of students who can enter the educational system. This problem can be studied with the aid of a very simplified model. It will be assumed that there are two educational levels, i=1,2. For these two levels the basic identities reduce to one variable, g_t^i, assuming that

$$n_t^i = s_t^i = g_t^i$$

and
$$r_t^i = d_t^{hi} = m_t^{hi} = 0 \quad \text{for } h = 1,2 \quad i = 1,\ldots\ldots\ldots,I.$$

The notation used is as follows:

g^i is the number of students in level i (all the students graduate);

T^i is the number of teachers in level i;

$\mu_T, \eta^i, \omega, \rho^i$ are parameters whose meaning will be clear from the equations.

The demand for teachers is given by:

II-2-4-1 $$T_t^1 = \rho^1 g_t^1$$

II-2-4-2 $$T_t^2 = \rho^2 g_t^2$$

while the supply of teachers is given by:

II-2-4-3 $$T_{t+1}^1 = (1-\mu_T)\ T_t^1 + \eta^1 g_t^2$$

II-2-4-4 $$T_{t+1}^2 = (1-\mu_T)\ T_t^2 + \eta^2 g_t^2$$

i.e., it is assumed that graduates from level 2 only can become teachers

Finally,

II-2-4-5 $$g^2_t = \omega g^1_{t-1}$$

where ω is the proportion of level 1 students entering level 2.

The above system is over-determined, having four unknowns and five equations. Since all the equations seem reasonable from an empirical point of view, the first conclusion is that the assumption of linear is too restrictive. In reality, relations like those above probably do determine the evolution of the educational system, but some flexibility is likely to exist in the parameter.

In order to understand the forces actually coming into play three different cases will be considered with the aid of the above equations.

In the first case it is assumed that the supply and demand of level 2 teachers, and its relation to the number of students, determine the evolution of the educational system. The system is centered on equations 4-2 and 4-4. Any one of the other equations is left out. In this case the evolution of the system is determined by

$$T^2_{t+1} = \left(1 + \frac{\eta^2}{\rho 2} \mu_T\right)^i T^2_o$$

i.e., the system will follow a path or steady growth.

In the second case it is assumed that the supply and demand of level 1 teachers and its relation to the number of students, determine the evolution of the system. The model is centered on equations 4-1, 4-3 and 4-5, and any one of the other two can be omitted. The evolution of the system is now determined by

$$T^1_{t+1} = (1-\rho_T)\, T^1_t + \eta^1 \frac{\omega}{\rho\ 1}\, T^1_{t-1}.$$

Depending on the values of the parameters, oscillations may appear.

In the third case it is assumed that g_t^1 is an exogenous variable. Two equations of the system must be left out. It is easy to check that a steady growth of g_t^i will initiate a steady growth of the educational system.

A certain extension of the above model would be very interesting. The above model shows that, to a certain extent, the educational system reproduces itself. This is true not only in a numerical sense, but also in an intellectual one; good teachers produce good graduates who will later on become good teachers. The opposite is also true. A model like the one introduced before could help in studying reforms of the educational system introducing new teachers and new methods. It will permit us to determine the minimum number of new teachers who could take over the whole educational system at some future date.

II-3 Costing and financing educational plans

II-3-1 Introduction

Two problems will be considered in this section: that of estimating the expenditure required to attain the targets of an educational plan, and that of estimating the resources available for financing education.

II-3-2 Expenditure required to attain the targets of an educational plan [23]

The targets of an educational plan can be and usually are, expressed or can be expressed in terms of the number of students that each educational process should have in a given year; that is, a target is expressed as a factor $[s_t]$ with components s_t^i, $i = 1,\ldots\ldots,I$.

The expenditure required to attain such targets can be classified as current or as capital. The estimation of the current expenditures required to attain the targets is given by the product

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[23] See references 14, 22, 39.

$$\left[c_t^1\right] \left[s_t\right]$$

where $\left[c_t^1\right]$ is a I - dimensional vector with components, c_t^{1i} representing the unit cost for students of educational process i; or a, I x I matrix with c_t^{1i} in the main diagonal. In the first case the total current expenditure would be obtained, in the second a vector of the expenditure by level.

Capital expenditure can be estimated with the product

$$\left[c_t^2\right] \left[s_t - s_{t-k}\right]$$

where $\left[c_t^2\right]$ is an I x I matrix with components c_t^{2i}, equal to the capital cost per student.

If convenient, the formulae above can be disaggregated, and estimated current costs given as the total of, say, administration costs, teacher costs and materials. In the same way, capital cost could be the total of building costs, laboratory costs, library costs, etc.

For administrative and accounting purposes, this disaggregation might have been continued to a very detailed stage.

For example, for estimating the expenses per pupil in Norway,[24] a model of 19 equations, 19 endogenous variables and 21 exogenous ones has been used. In this model the cost per pupil is a total of 10 components.

To estimate cost when disaggregate information is used, the above formulae must be modified, although detailed explanation of these modifications is not required here.

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[24] See reference 33.

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The main problem in estimating the expenditure required to attain the targets of an educational plan is the evaluation of the components of the vectors $\left[c_t^{hi} \right]$, $h=1,\ldots,H$; $i = 1,\ldots,I$, where h is the type of cost, and i the educational process. Furthermore assessing the future evolution of teachers' salaries is a major difficulty in estimating this expenditure. No attention has been given to this problem.[25]

II-3-3 Estimation of the resources available for education[26]

In the study of resources available for education, techniques similar to those used for the study of expenditure in consumption with respect to total income have been used. Three models have been suggested:

$$E_t = \varepsilon^1 \, Y_t^{\phi^1}$$

$$e_t = \varepsilon^2 \, y_t^{\phi^2}$$

$$E_t = \varepsilon^3 \, Y_t^{\phi^3} P^{\theta^3}$$

where
E = resources available
Y = National production
P = Population
$e = \frac{E}{P}$
$y = \frac{Y}{P}$
ε^i, ϕ^i and θ^i constants.

The above models can be disaggregated, for example, in order to separately consider the total private resources available for education.

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[25] See references 12 - 14.

[26] " " 3 - 14.

As an extension, the total resources (both public and private) could be used as an independent variable to estimate the foreign aid available for education.

Such problems as the possible distribution of the total resources likely to be available among different educational levels have not been considered.

II-4 Educated personnel required for social development

II-4-I Introduction

Models determinating the characteristics that the educational system must have in order to achieve the targets set for its output were presented in sections II-1-4-2 and II-1-4-4. Nothing was said about the method that should be used to determine such targets. In the present section no method will be given either. This is more a political than a technical problem.

In the targets of any educational plan, however, attention should be paid to the contribution of education to social development. This means that the targets of an educational plan must include the educated personnel required for social development. In this section some models used to determine such needs of educated personnel will be presented.

First, the problem of the educated personnel necessary for one aspect of social development, that of economic development, will be studied. The methodology in this case is more advanced. Next, some remarks will be made about its generalization to other aspects of social development.

II-4-2 Manpower needs[27]

The demand functions for qualified labour that have been proposed below.

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[27] See references 10, 12, 14, 19, 26, 32, 41, 44.

II-4-2-1 $$L_t^i = \lambda^i Y_t$$

II-4-2-2 $$L_t^{ij} = \lambda^{ij} Y_t^j$$

II-4-2-3 $$L_t^i = \nu^{io} Y_t^{\nu^{i1}} \cdot Y_t^{\nu^{i2}}$$

II-4-2-4 $$\ell_t^i = \sigma^{10} \ell_t^{\sigma_t^{i1}} \quad \text{with} \sum_i \sigma_t^{i1} \frac{\ell_t^i}{\ell_t} = 1$$

where L^{ij} is labour with educational level i, economic sector j;

Y^j is the output of economic sector j;

$$L^i = \sum_j L^{ij}$$

$$L = \sum_i L^i$$

$$Y = \sum_j Y^j$$

$$y = \frac{Y}{P}$$

$$\ell^i = \frac{L^i}{Y} \; ; \quad \ell = \frac{L}{Y} = \sum_i \ell^i$$

Σ and σ are constants.

In functions (2-1) and (2-2), qualified labour and production are related by a constant of proportionality. If required, function (2-2) could be modified to relate qualified labour to total production determined by input and output methods.

In function (2-3), the demand for qualified labour is related through constant elasticities to total and per capita production.

In this function, it is recognised that one of the main reasons why qualified labour is demanded is that labour productivity has increased. This factor is introduced indirectly by means of per capita income.

Function (2-4) belongs to a larger model. In this model Y and L are assumed to be exogenous variables. These exogenous variables permit us to determine

$$\ell = \frac{L}{Y} .$$

With this value and equation 2-4, the values of σ^{i} can be found. The condition imposed upon the elasticities insures that

$$\sum_{i} L^{i}_{t+} = L_{t+}$$

i.e., a natural condition is fulfilled.

It should be observed that the σ^{i1}_{t} are estimated in iterative steps. The basic statistical data permits us to estimate σ^{i1}_{o}. With this, and the known values of L_1 and Y_1, the values of ℓ^{i}_{1} can be estimated. With these data the value of σ^{i1}_{1} can be obtained, and so on.

Several extensions of the above models are possible. In particular it would be interesting to explicitly introduce data on salaries, and to study problems of substitution between different levels of qualified labour. All the models of demand for production factors can be applied to the study of the demand for qualified labour. Here is one case in which the limitation is not in the models but in the data.

In the models just presented no mention is made of the fact that the labour force is not usually classified by level of education, but by occupation. There is no mathematical problem in passing from occupation to level of education or vice versa. For this a matrix of transformation is used. The main point, however, is that occupations are defined in terms of what the person does, and so their study might help us to find the knowledge required by a worker. This aspect is important when the content of education is planned.

In actual studies of the relation between production, manpower and education, the form of the mathematical relations might not receive attention. Trial and error methods to obtain estimates that are intuitively acceptable have been recommended and applied.[28] To use such methods large computing facilities are necessary particularly if disaggregated information is employed.

Another factor to be considered in the determination of manpower needs is that people move from one occupation to another during their active lives. Thus experience and on-the-job training could be explicitly introduced in the models for estimating manpower needs.

So far only the case of demand for qualified labour as a factor of production has been considered. Two extensions should be considered. One is that qualified labour can be demanded as a final product, for example, in the case of demand for services such as of doctors, dentists, etc. In this case the model for the demand of final products should be applied. In the second case, educated people are demanded not by economic processes, but by other social processes for example, by public administration. This brings us to the second question: demand of the educated persons for other aspects of social development.

When the number of workers with different levels of qualification is known, it is possible to estimate the educational structure of the population that would supply the labour force with the required qualifications. This could be done by dividing the number of workers by the participation rates of different levels of qualifications. This information, and the methods described in sections II-1-4-3 and II-1-4-4, would permit us to determine the characteristics that the educational system should have in order to produce the educational structure of the labour force required to achieve the targets of the economic plan. As was mentioned previously, these characteristics are only one of aspects that should be considered in determinating the targets of the educational plan.

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28. I will not make any attempt here to give all the references on this topic. Valuable references on this topic are 4 and 5.

II-4-3 Generalization of the model

When the approach used to study manpower is carried to its logical conclusion we are forced to face a contradiction. Most people in a society are "used" for economic development. If this is so, no other aspect of social life could be attended to, or, if these other aspects are considered a larger population is required.

The reason for this contradiction is that in the analysis of manpower needs the difference between people and roles played by people has been overlocked. An economic occupation is only one of the roles played by a person in a society. Only as a simplification is it possible to say that a certain number of persons are needed for economic development. Actually, to obtain a certain volume of production certain roles must be played at certain times. When we discover how to make machines to play these roles, machines can be used instead of persons.

When estimating manpower needs, what should be estimated is the number of hours for which the economic roles are played. This change of point of view, from number of persons to number of hours during which a role is played, is important when several social functions are considered in determinating the number of educated people required for social development.

To begin with, we should estimate the time during which roles in, say, health, the family, polity, education and the economy must be played in order to attain the targets in these areas. The main problems are to define the roles in each of these areas of social life, and to measure the time devoted to them.

The next problem is to pass from time for which roles are played to persons. For this it should be observed that each person not only plays roles in health, the family, the economy, education and polity, but also that the roles of each person are interrelated in what can be called a constellation of roles. If a refined classification of roles were made, each constellation of roles would be the daily schedule of the life of a person. Of course, in practice, aggregate approximations would have to be used.

This analysis should include the fact that the constellation of roles changes with social development, and that a person changes roles during his life.

Some aspects of the model just outlined could be qualified, and a mathematical model could be used to study them. For example, Markov chains could be used to study the progress from one role to another.

The number of hours for which the different roles are played could be used to determine the number of hours spent learning those roles. In this way, the schools' curriculum could be planned. This information, and constraints on the number of students, would give the length of school life. Alternatively the information about the roles to be learned, the time required to do so and the time available, would permit the determination of the different specializations to be studied.

It seems to me that only this generalized approach, based on roles, constellation of roles and social functions, will permit the passage from manpower planning to planning the demand for educated personnel for social development. The remarks above are just some initial thoughts that, I hope, will provoke further thinking and experimentation.

III. MODELS WITH CHOICE AMONG ALTERNATIVES

Several mathematical techniques to choose the optimum among possible alternatives have been applied to educational planning. Linear programming is the technique most frequently used. For this reason, only those situations where linear programming can be applied will be considered below.

III-1 An optimum enrolment policy

Most developing countries are committed to a policy of general education for all persons of school age. This means that they should open schools not only for those reaching six years of age, but also for all persons of 7,8 .... years of age who have not enrolled in the past.

A country that in one year manages to open schools for all the back-log of non-enrolled people will waste its resources. This is so because the school facilities made available for the back-log of non-enrolled will be under-utilised when, in the future, only those reaching six years of age enrol in the educational system.

If the enrolment of the back-log of non-enrolled people is spread over several years, it is possible to avoid this wastage of resources. Below, the question of the minimum investment needed to enrol the back-log of students and people reaching six years of age is put in the form of a linear programming problem.

Let us suppose that in year t=0 a country has a back-log of non-enrolled persons of

$$\sum_{h=1}^{H} \bar{P}_{oh}$$

where $\bar{P}_{oh}$ is the number of non-enrolled persons of age h in year t. It will be assumed that $h=o,\ldots\ldots\ldots,H$, with $h=o$ the age of the youggest students, and $h=H$ the age of the oldest new entrants in the educational system.

In addition, let us suppose that the country wants to open schools for all the back-log in the period between $t=o$ and $t = \gamma$. It will be assumed that $\gamma > H$.

The following equalities (in which deaths have been eliminated) must be fulfilled in order to open schools for all those in the back-log :

III-1-1
$$\sum_{j=o}^{H-h} \bar{n}_{j;\ h+j} = \bar{P}_{oh} \qquad \text{for } h = o,\ldots\ldots,H$$

where $\bar{n}_{t;h}$ is the number of new entrants in the educational system in year t and of age h.

Let us further assume that the country wants to have schools for all those reaching six years of age between $t = 1$ and $t = \gamma$;i.e.,

III-1-2
$$\sum_{j=o}^{J} \bar{n}_{t+j;\,j} = P_{t,o} \qquad t=1,\ldots\ldots\ldots,\gamma$$

with
$$J = \begin{cases} H & \text{if } \gamma - t \geqslant H \\ \gamma - t & \text{otherwise.} \end{cases}$$

Equations 1-1 and 1-2 refer to all the back-log and all the persons reaching six years of age ; however, if required they can be modified to include a known part of either or of both of them only.

At any time t the total number of new entrants in the educational system will be

III-1-3
$$\bar{n}_{t\cdot} = \sum_{h=o}^{H} \bar{n}_{t,h} \qquad t=o,\ldots\ldots\ldots,\gamma .$$

The problem here is to minimize the investment required to support $\bar{n}_t$, for $t = o, \ldots\ldots\ldots, \gamma$. Let us assume that schools for $\bar{n}_{-1}$ students are available, i.e.

$$K_{-1}^2 = c_0^2\, \bar{n}_{-1}$$

where K_t^2 is the capital in education in year t, and

c_0^2 is the capital per student ratio.

For the students entering in year 0 total capital required - assuming no depreciation - will be

$$K_{-1}^2 + I_0 = c_0^2\, \bar{n}_0$$

where I_t is investment in year t. For those entering in year 1 the total capital required will be

$$K_{-1}^2 + I_0 + I_1 = c_0^2\, \bar{n}_t$$

and in general

$$K_{-1}^2 + I_0 + I_1 + \ldots\ldots + I_t = c_0^2\, \bar{n}_t$$

for $t = 0 , 1 , \ldots\ldots, \gamma$.

The problem now reduces to minimizing total investment

$$Z = \sum_{t=0}^{-1} \bar{c}_t^2\, I_t$$

where $\bar{c}_t^2$ is a depreciation factor of investment in year t, subject to conditions 1-1, 1-2, and 1-3, 1-5 and 1-6, and to $\bar{n}_t \geqslant 0$.

III-2 Flows of students, teachers, buildings and costs [29]

The basis of the decision models referring to flows of students, teachers, buildings and costs are the identifies introduced in section II-1-2. To consider the most general case, the aggregate version of these identities will be used. It will be assumed that there are k years between two consecutive pivotal years, and a total of $t = 1$ pivotal years will be considered, i.e., $t = o, k, 2k, ..$. We will also assume, as before, that there are I educational levels, i.e., that $i = 1,....,I$. Finally, the numbers of students in the first and last pivotal years are assumed to be known, i.e., s_o^i and s_γ^i are exogenous data.

We will now proceed to the presentation and description of the model. The problem is to maximize a weighted function of the elements in the output of the educational system, i.e., to maximize

III-2-1
$$Z = \sum_{t=o}^{\gamma-1} \sum_{i=1}^{I} \left[\xi_t^i \, g_t^{1i} + x_t^i \, d_t^{2i} + \zeta_{t+k}^i \, d_{t+k}^{1i} \right]$$

where ξ^i, x^i and ζ^i are weights given to different elements of the output of the educational system.

Different quantities could be used as weights in the objective function (III-2-1). Reasonable possibilities are the economic (private and/or social) returns of education and the number of years of education. In the first case, Z would be total economic return, and in the second, total number of years of education. Of course, the weights just described could be modified by taking into consideration preferences of the policy makers. For example, the weights could be modified to include the fact that a policy maker considers that a year of general education is equivalent to one and a half years of vocational education. There are no limits to this type of modification.

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[29] See references 7, 11, 13, 16, 17, 20.

However, the basic obstacle in determining an objective function for education is not in the choice of a set of weights which can be used in a particular case. Any choice will be arbitrary. These remarks carry us to the heart of the problem: very little is known in economics of the characteristics of the preference functions. These functions are the basis of all the economic analysis of choice. Additional knowledge in this area would perhaps help to solve the problem of the objective function for education.

It should be observed that no mention was previously made of the problem of constructing an objective function of social preferences with respect to education. But even if all the problems of constructing a social preference function, taking the individual preference function as a starting point, were solved, I do not believe that we could not formulate such a social preference function. The basic problem is that we do not know how to construct an individual preference function starting from sample surveys of psychological tests.

The first type of constraints that should be considered are the basic identities describing the student flows; i.e., the maximisation should be subject to:

III-2-2 $$s_t^i = n_t^i + r_t^i - d_t^{1i} - m_t^{1i}$$

III-2-3 $$s_t^i = g_t^i + r_{t+k}^i + d_t^{2i} + m_t^{2i}$$

with s_o^i and s_γ^i known.

For the new entrants the following constraints should be considered:

III-2-4 $$n_t^i = \sum_{j=1}^{I} n_t^{ij} + \sum_{j=1}^{t-1} \hat{n}_{t-j}^i; \quad i = 1,\ldots\ldots\ldots,A$$

III-2-5 $$n_t^i = \sum_{j=1}^{I} n^{ij} \qquad i = A+1,\ldots,I$$

where n_t^{ij} the number of new entrants in educational process i, structure t, coming from process j, structure t-k; and

$\hat{n}^i_{t;j}$ the number who can enter educational process i, from two sources: those just reaching school age, and those who could have entered in the past but did not do so. The subscript t refers to the current year; the subscript j to the year they could have entered.

The first of the equations above refers to the new entrants in levels 1 to A, that is in those levels open to persons with or without previous education; while the second refers to the levels open only to those with previous education.

n_t^{ij} is subject to the following constraints:

III-2-6 $$\sum_{i=1}^{I} n_{t+1}^{ij} \leq g_t^j \qquad j = 1, \ldots\ldots, I.$$

That is, the total number of people with educational level j who continue their studies must be at most equal to the number of graduates from that level. Knowledge of a particular educational system allows us to put

III-2-7 $$n_t^{ij} = 0$$

for those levels, j, from which it is not possible to pass to level i.

For the levels of education open to persons with or without education those coming directly from the population should be considered. The analysis in section II-1-4-3 permits us to determine the maximum possible number of new entrants at t=o and of those reaching school age in periods t=1 , ...Y . As before

$\bar{P}_{t,h}$ is the number of non-enrolled persons, age h, in period t; and let

$$\bar{P}_t = \sum_{h=1}^{H} \bar{P}_{t,h}.$$

Not only those reaching school age in year t are included in $\bar{P}_t$, but also all those who should have enrolled before, but did not do so, and are still suitably aged to enrol in a school.

With the notation just introduced, the following constraint for $\hat{n}^i_{t;j}$ can be set:

III-2-8 $$\sum_{i=1}^{A} \hat{n}^{i}_{t;j} \leq \bar{P}_t - \sum_{h=o}^{j-1} \sum_{i=1}^{A} \hat{n}^{i}_{t,h}.$$

Another natural constraint to be imposed is that all the variables must be larger or equal to zero.

Several alternative assumptions can be made about the relations among the variables in the basic identities. To begin with, it is reasonable to assume that

III-2-9 $$m^{1i}_t = \mu^{1i}_t \left(n^i_t + r^i_t\right)$$

III-2-10 $$m^{2i}_t = \mu^{2i}_t \, s^i_t$$

i.e., that is the number of deaths determined by a mortality rate and cannot be determined as a policy variable.

The cases of drop-outs and repeaters[30] do not have such clear cut solutions. It would be possible to leave these variables free inside the limits imposed by the conditions set above and those affecting all the variables to be introduced below. A second possibility is to limit the variation of drop-outs and repeaters. For instance, constraints such as

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[30] See reference 11.

III-2-11 $$\delta^{1i}(n_t^i + r_t^i) \leqslant d_t^{1i} \leqslant \delta^{1i}(n_t^i + r_t^i)$$

could be introduced. The interest of these constraints is that they are related to the quality of education. In practice it would be advisable to consider both the unrestricted and the restricted values of the numbers drop-outs and repeaters. The first case would set what is desirable, the second what is possible.

When the restricted variation of drop-outs and repeaters is considered, the values of the parameters, say $\bar{\delta}^{ij}$ in the example above, should be related to different educational technologies expressed in terms of teachers, buildings, costs, etc.. Additional comment on this problem will be made later.

The equations introduced so far correspond to the simplest problem of choice between flows of students. These equations are the counterpart of the problem considered in section II-1-4-2. In that section, previously existing conditions determined the future enrolment and the output of the educational system. In the present case, the time sequence of enrolment in the future and the output of education are determined in such a way as to obtain optimum results. However, no attention has so far been paid to the problems of the human or physical resources required in education. These aspects will be introduced below.

To introduce teachers[31] in the above model, let

D_t^{hi} be the number of graduates from level i becoming teachers in level h;

η^i be the proportion of graduates from level i leaving the education system to graduates becoming teachers;

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[31] 31. See reference 11.

$$D_t^{\cdot i} = \sum_{h=1}^{I} D_t^{hi}$$

Equation III-2-6 must first be modified in the following way:

III-2-12 $$g_t^i \geqslant \eta^{i} D_t^{\cdot i} + n_{t+k}^{\cdot i}$$

i.e., the graduates must provide both teachers and new entrants at higher levels. If drop-outs can also become teachers, constraints taking this fact into consideration could be included.

The evolution of the number of teachers in level i is given by

III-2-13 $$T_t^i = T_{t-k}^i (1- \mu_T) + \sum_{j=1}^{I} D^{ij}$$

Furthermore, the size of the student body depends on the number of teachers available, i.e.,

III-2-14 $$s_t^i \leqslant \rho - T_t^i$$

where ρ is the relation between numbers of students and teachers introduced in section II-2-3.

As mentioned above, it is possible to consider qualitative aspects of the model being studied. There are several ways to do so. One is to consider alternative values for ρ^i, and to relate them to alternative constraints for drop-outs and repeaters. Another possibility is to reduce to a minimum the numbers of drop-outs and repeaters when teachers are graduates only, and to increase these numbers when teachers need not be graduates. Another way of introducing these qualitative aspects is to consider that the output of schools with highly qualified teachers has a larger weight in the objective function than the output of other schools. These two methods could be used simultaneously.

Following lines similar to those used above, the availability of buildings and others facilities could be introduced. In particular, the problem of optimum size and location of schools could be studied.[32] However, these matters are rather closely related to financial resources, and only this aspect will be considered in some detail below.

Let us suppose that E is the amount of resources available for education during the period between $t=o$ and $t=Y$. Let

c^{hi}_t	be the cost per student level i and if h=1, the current cost during a period of k years.
K^{2i}_t	be the capital available for educational level i, period t.

Then the following constraints can be considered:

III-2-15 $$K^{2i}_{t+k} = K^{2i}_t (1-\mu_K) + \Delta K^{2i}_{t+k}$$

III-2-16 $$c^{2i}_t \; s^i_t \leq K^{2i}_t \quad \text{and}$$

III-2-17 $$\sum_{t=o}^{Y} \sum_{i=o}^{I} \left[c^{1i}_t \; s^i_t + \Delta K^{2i}_t \right] \leq E.$$

III-3 Allocation of resources between the economy and the educational system[33]

Another problem that can be studied with the aid of the techniques of linear programming is that of the allocation of resources between the economy and the educational system. This question can be considered as an extension of the one treated in Section III-2, and it is with respect to the constraints that the variables must be satisfied. However, while in section III-2 a fairly acceptable objective function could be determined in the present case, such a function is difficult to obtain.

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[32] See reference 16.

[33] See reference 1, 9, 43.

To simplify the presentation we will first introduce the constraints, and then some considerations about the objective function.

As an initial example we will consider the economy aggregated in one sector.

Let Y_t be the G.N.P. between t-k and t;

C_t be the consumption between t-k and t;

K_t^1 be the capital in the economy;

K be the total capital; i.e., capital in the economy plus capital in the educational system.

With this notation the basic economic equations are

$$Y_t = C_t + \Delta K_t$$

$$\Delta K_t = \kappa Y_t$$

$$\kappa^1 Y_t = \Delta K_t^1$$

$$K_t^1 = K_{t-k}^1 (1- \mu_k) + \Delta K_t^1$$

$$Y_t \leqslant \frac{1}{\lambda^i} L_t^i .$$

The second step in the construction of this model is to relate qualified labour to the educational structure of the population. This is done with the following equation:

$$L^i_t + T^{\cdot 1} \leqslant \pi^i G^i_t$$

where $T^{\cdot i}$ is the number of teachers with educational level i, and

π is the ratio participation in the labour force.

In the next step the relation between the educational structure of the population and the output of the educational system is described. The evolution of the educational structure of the population is given by

$$G^i_t = G^i_{t-k} \ (1 - \mu^i_G) + (1 - \mu^i_o) \ O^i_t.$$

Next we have $O^i_{t+k} = g^{1i}_t + d^{2i}_t + d^{1i}_{t+k}.$

Then we proceed to the analysis of the educational system. For this the equations III-2-2 to III-2-16 and observations in Section III-2 are used, and constraint III-2-17 is dropped.

A final constraint is required relating the total amount of resources for investment to the investment in the economy and the educational system. This constraint takes the following form:

$$\Delta K^1_t + \sum_{i=1}^{I} \Delta K^{2i}_t \leqslant \Delta K_t.$$

This constraint, in this presentation of the model, is equivalent to constraint III-2-17 in the model in section III-2.

To complete the analysis of the constraints it should be observed that the aggregated analysis of the economy can be replaced with a disaggregated approach. Since disaggregated linear economic models are familiar we will not go into details.

In the case of the allocation of resources between the economy and the educational system, the main problem is that of the objective function. In this case, as in Section III-2, there is the possibility of giving arbitrary weights to the economic and educational variables and of constructing an objective function with such weighted variables. But again, there is no possibility of justifying the choice of weights. The reasons given in Section III-2 are also valid here.

It should be observed that the special cases of the objective function just described are those in which the educational variables or the economic variables have zero weights. In the first case only the economic variables would be maximized, and in the second only the educational variables. For instance, in the first case an acceptable choice would be to maximize

$$C = \sum_{t=o}^{\gamma} \bar{C}_t$$

where $\bar{C}_t$ is the present value of consumption in period t. In this case the value of ΔK_γ must be determined exogenously. If this is not done the model requires that $\Delta K = 0$. If only educational variables are maximized, a function such as III-2-1 is considered. In this case the weights of the objective function must have an educational meaning.

The two special cases described above permit us to introduce another type of objective function, namely, to approach both maxima mentioned above as closely as possible. This can be done in the following way. Let us make

$$C_o = \sum_{t=-\gamma}^{o} \bar{C}_t$$

i.e., let us make be the actual consumption in the past $\gamma + 1$ years; and

$$Z_o = \sum_{t=-\gamma}^{o} \sum_{i=1}^{I} \left[\xi_t^i \, g_t^{1i} + x_t^i \, d_t^{2i} + \zeta_{t+k}^i \, d_{t+k}^{1i} \right]$$

i.e., Z_o is the actual value of the objective function for education in the past $\gamma + 1$ years. It can be said that C_o and Z_o describe the present situation as it is.

Let $\bar{C}$ and $\bar{Z}$ be the maxima described above; i.e., $\bar{C}$ is the maximum of the economic variables only, and $\bar{Z}$ of the educational variables only, subject to the constraints introduced above. In this case $\bar{C}$ and $\bar{Z}$ represent the optima that could be obtained.

Consider now a variable, x, that takes the values zero for C_o, and 1 for C. In this case a linear function of x passing through the points $(0,C_o)$ and (1, C) can be established. With the same variable, x, a function through the points $(0, Z_o)$ and (1, Z) can be determined. Now it is possible to pass from the present conditions, C_o and Z_o to the best possible conditions $\bar{C}$ and $\bar{Z}$, maximizing the value of x. This is equivalent to approaching as closely as possible both $\bar{C}$ and $\bar{Z}$.

III-4 Optimum educational curriculum[34]

Some of the links between quantitative and qualitative aspects in educational planning were pointed out in section III-3. Another link is indicated in this section where a model to determine an optimum educational curriculum is presented.

In preparing the curriculum of an educational system or of an educational institution, two different operations must be performed.

The first corresponds to the work of educationists, psychologists and scientists in all branches of human knowledge. They must begin by dividing human knowledge into fields or subjects. A list from this division will come, beginning with, say, astronomy, and ending with zoology. These subjects must further be divided into courses, such as elementary, intermediate and college algebra, or Latin American, European, African history, etc. We will assume that such courses are the building blocks of any curriculum.

Once the list of courses is complete, the teachers of the different courses can prepare lists of prerequisites. The meaning of this should be clear to any one who has read a catalogue from an American university. It will be found, for example, that in order to enrol in Physics 210, one must already have taken Physics 209, as well as Mathematics 180. There is no reason to stop there. There are also prerequisites for entering university and for entering high school. Thus, for each course, a list of prerequisites can be prepared. These lists would end with the first courses taken in elementary school.

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[34] See reference 15.

The first operation in the preparation of a curriculum ends with the preparation of lists of prerequisites for each course. All the courses that can be taught and their prerequisites are now known. Any course might be a prerequisite for some other; and any course might be final, in the sense that some students might not take any additional courses on the same subject.

The second operation in the preparation of a curriculum is as important as the first. It is the actual decision of which courses should be included in the curriculum, for it is obvious that all the possible courses and all their prerequisites could not possibly be included. The lives of the students and teachers are too short, and the money available to the schools is too scarce, to allow for the teaching of all possible courses. Thus, in the preparation of a curriculum there is a problem of choice.

In order to decide which courses should or should not be included, educationists must compare the different courses with other courses, and with their costs in scarce resources.

This problem does not differ from that of applying mathematical programming to, say, optimizing the output of an industry. In this case, industrial engineers delimit the technological alternatives available; and monetary costs and prices usually provide the means for comparing the benefits and the costs in resources of the different techniques. In both education and industry, the problem is to optimize the use of resources.

The problem just described will be written below in mathematical notation. The first element provided by the educationists is a list of courses and/or prerequisites. Let z^i be a course, say, three semester-hours of physics;

$$i = 1, \ldots\ldots, a.$$

For each course, the educationists will have a list of prerequisites. From these lists a (x) matrix $[z]$ can be formed in the following way. The components z^{ij} will be

z^{ij} = 1, if subject j is a prerequisite for subject i;

z^{ij} = 0, if it is not;

z^{ii} = 1, by convention.

The matrix $[z]$ has only zeros and ones as components. From any of its rows, say z^{hj}, the prerequisites of subject z^h can be read. They appear as a number one in the column of the prerequisite subject, z^i. The matrix $[z]$ is a systematic description of the teacher's lists of prerequisites; and adds nothing to them.

As was mentioned before, the educationist must know the benefits and costs of the different courses. We will assume here that only one scarce resource exists. Let

o^i be the cost of teaching course i = 1,......,n;

ζ^i be the benefits of course i.
Benefits of prerequisites are not included, That is, if z^j is a prerequisite of z^i, the total benefits of taking z^i will be $\zeta^j + \zeta^i$.

E is the amount of resources available.

The matrix of prerequisites $[z]$, the vectors of cost and benefits $[c]$ and $[\zeta]$, and the quantity of resources Z is the data that educationists must prepare. Let us see what can be done with it.

Let (1) be an m-dimensional vector with all its components equal to one;

a^i be the number of prerequisites for subject i; and

$[a]$ be the dimensional vector with a^i as components.

It can be seen that

$$[z]\ (1) = [a]$$

Our problem is to decide whether z^i should or should not be included in the curriculum. In order to do so, let us consider the variables

$$x^i = 1, \text{ if course } z^i \text{ is taught}$$

and $$x^i = 0, \text{ if it is not taught.}$$

The fact that x^i takes only the values 0 or 1 can be expressed by means of the relation

$$x^i = (x^i)^2$$

where $(x^i)^2$ is the square of x^i.

The relation between the different courses and prerequisites can be established in the following way: in order to teach a course, say s^h, its prerequisites must also be taught. This means if $x^h = 1$, the following equality will hold:

$$(z^{hj})\ (x^j) = a^h$$

in which (z^{hj}) is the h-row of matrix $[z]$, and (x^j) is a column vector with x^j as components, because this equality is possible only if the x^j corresponding to the prerequisites of x^h are taught.

On the other hand, all the prerequisites of z^h could be taught, but not z^h itself. Thus, x^h could be zero while x^j corresponding to a prerequisite has a value of one. From this analysis, we can summarize the relationship between courses and prerequisites by

$$a^h x^h \leqslant (z^{hj})\ (x^j).$$

If z^h is taught, $x^h = 1$, and the equality sign must prevail. If it is not taught, and some prerequisite is taught, the inequality will prevail. Finally, if neither z^h nor its prerequisites are taught, equality will prevail. Both sides of the relation will be zero.

The relation between cost and resources can be expressed :

$$(c^i) \quad (x^i) \quad \leqslant \quad E$$

where (c^i) is a row vector with c^i as components. Finally, total benefits will be

$$Z \; = \; (\zeta^i) \quad (x^i)$$

where (ζ^i) is a row vector with ζ^i as components.

These relations permit us to express the curriculum problem as follows :

Maximize III-4-1 $Z \; = \; (\zeta^i) \quad (x^i)$

subject to III-4-2 $a^h \, x^h \leqslant (z^{hj}) \quad (x^j)$

III-4-3 $x^j \; = \; (x^j)^2$

III-4-4 $E \geqslant (c^i) \quad (x^i)$

To solve this problem observe that all the 2^n vectors that can be formed by putting 0 or 1 for each of the components of the vector (x^j) are the extreme points of a cube with unit side in the η-dimensional euclidean space. For instance, if $n = 2$, the 2^2 vectors are (0,0), (1,0), (0,1) and (1,1), that is, the extreme points of a square with unit side.

From this observation, we can conclude that the problem formed by equations 4-1, 4-2 and 4-3 above has the same solution as the problems in which condition 4-3 is changed to

III-4-5 $0 \leqslant x^j \leqslant 1.$

This is so because in equations 4-1, 4-2 and 4-5 we are considering a a linear programming problem defined over the convex hull of the problem in equations 4-1, 4-2 and 4-3. Furthermore, the extreme points of the convex hull are exactly those acceptable as solutions to the problem in 4-1, 4-2 and 4-3. Thus, in this case, the usual technique for solving linear programming problems can be used.

So far, we have not considered condition 4-4 in the problem in equations 4-1 to 4-4. When this is done, two cases are possible.

4-6 $$(c^i)(1) < E,$$

that is, if the resources are not scarce, the hyperplane

4-7 $$(c^i)(1) = E$$

does not intersect the n-dimensional unit cube. No new extreme points appear, and the solution of 4-1, 4-2 and 4-3 obtained by the usual linear programming techniques is that of 4-1 to 4-4.

In the second case, the inequality in 4-6 is reversed. The hyperplane in 4-7 intersects the n-dimensional unit cube and new extreme points appear. These new extreme points might have co-ordinates with some components which differ from zero or one. That is, for these new extreme points we will have

$$0 < x^j < 1$$

for some j. To solve this problem use must be made of the techniques for finding solutions to linear programming problems in which the values of the variables are integers. Again, the solution of the problem in equations 4-1, 4-2, 4-4 and 4-5 is that of the problem in equation 4-1 to 4-4. Therefore, in summary it can be said that a method is available for solving the curriculum problem as stated in this section.

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NOTATION

c	costs c^i $i = 1$ current $i = 2$ capital
C	consumption
d	drop-outs
	d^1 before
	d^2 after
	d^{1ij} drop-outs before ... from level j considered as output from level i
	d^{2ij} drop-outs after ... from level j considered as output from level i
e	per capita resources available
E	total resources available
f	function (....)
g	graduates
	g^i those graduates from one level who leave the educational system
G^i	non-students in the population with educational level i
H	age
k	years of aggregation for school system
K	matrix defined on page 30
K^i	capital, $i = 1$, in the economy $i = 2$, in education
l	number of workers per unit of output
l^i	number of workers with education level i

L	total number of workers.
m^i	deaths
	m^1 : before t
	m^2 : after t
M	period offered
n	new entrants in an educational structure
$\bar{n}$	new entrants in the educational system
n^{hi}	new entrants in level h coming from level i
$\hat{n}$	people remaining in the educational structure
N	periods of education received
o	output
P_s	periods per student
P_c	periods per teacher
P_r	periods per room
P	population
$\hat{P}$	stationary population
$\bar{P}$	non-enrolled population
R	rooms
s	students
$\hat{s}$	stationary school population
$\bar{S}$	average class size
t	time
T	number of teachers
v	re-entrants
y	per capita income
Y	total income

α	proportion of graduates $g^i = \alpha^i s^i$
β	proportion of inputs in previous structure $s^i_t = \beta^i(n^i_{t-k} + r^i_{t-k})$
γ^{ij}	proportion of graduates from j new entrants in i
γ^{ii}	proportion of repeaters in level i
$\gamma^{\cdot j}$	proportion of graduates from j who remain in the system
Γ	matrix defined on page 29
δ	proportion of drop-outs δ^{1i} drop-outs before δ^{2i} drop-outs after δ^{1ij} drop-outs before from level j, considered as output of level i
Δ	increment
ε	resources available for education
ζ	objective function
η	proportion of graduates becoming teachers
K	capital-output ratio
λ	labour-output ratio
μ	mortality rate
μ^i_o	mortality rate in output level i
μ^i_G	mortality rate in the population
μ_K	depreciation of capital
ξ	objective function
ρ	weighted student-teacher ratio
π	participation rate
σ	elasticity ℓ, see page 40

$\sum$	sumation sign
T	student-population ratio
Σ	defined on page 40
Υ	defined on page 45
X	objective functions
ω	component of Ω
Ω	matrix defined on page 28

PART III : ENROLMENT PROJECTION

PROJECTION MODELS OF THE SWEDISH EDUCATIONAL SYSTEM

by the Forecasting Institute of the Swedish
Central Bureau of Statistics

Background

For many years Swedish population statistics have been of relatively good quality. There has been a successive increase in the collection of statistical data on education. Thus, the quantity of collected statistics concerning students in higher education increased in 1937, 1956 and 1963. The present annual statistical programme for primary and secondary education was introduced in 1960-1961. Compared with most other countries, Swedish educational statistics are quite ambitious. However, regularly collected statistics regarding a few small sectors of higher education, and important sectors of vocational education, are still missing.

Swedish educational planning has been pursued mainly within the jurisdiction of ad hoc committees (often with members of Parliament) appointed by the Minister of Education. By and large these committees have used available statistics in their work, but have also collected statistical data of their own. As a basis for their proposals, the committees have often been commissioned to make projections regarding the supply and demand of persons with the particular type of education under investigation.

These ad hoc projections have often not been co-ordinated with projections for other sectors of education, and for the labour market as a whole. Furthermore, it has been questioned whether it is realistic to forecast demand for what are usually small and highly specialised branches of education or professions 10-15 years in advance.

In 1960 a working group was appointed within the National Labour Market Board. The group was charged with making projections for the supply and demand of different categories of education. This working group, the "Forecasting Institute", took part in making projections for several important parts of the labour force which were under investigation by special committees. Apart from this, a number of forecasts required by authorities or organisations were made. However, it was questioned whether it is possible to make long-run forecasts for specialised categories of education. The objective of the work of the Forecasting Institute has therefore been somewhat modified. Instead of making independent forecasts for small specialised groups the work is now aiming at a more integrated approach in order to comprise larger sectors of the labour market, and is to the highest degree possible to be co-ordinated with economic and social planning in general. As an element of this broader scope the Forecasting Institute has started work on models of education. In 1964 the Forecasting Institute was transferred from the National Labour Market Board to the National Central Bureau of Statistics.

Objectives of the work on models of education

Much has been written about what one would wish to gain from different models of education. The objectives of the work of the Forecasting Institute in this field are therefore summarised by only a few points.

(i) The models are supposed to illustrate the dynamics of the educational system, and to serve as a framework for the interpretation of statistics of education later on.

(ii) The models may be used for projections of observed trends possible, and to study the implications of decisions already made.

(iii) The models may be used to illustrate and analyse the effects of alternative decisions.

(iv) As the models, in principle, comprehend the educational system of the whole country at all levels, their purpose is also to obtain consistency between points (i), (ii) and (iii) in definitions and codes, and also to get co-ordination between the planning being pursued for different sectors of the educational system.

Choice of models

The choice of models is a choice of mathematical models to describe a certain structure. This choice is directed by the problems one wants to throw light upon, the supply of primary information (in the case of practical usage of the models) and wishes regarding the level of aspiration and richness of detail of the models. So far, the starting point for the work on models of education at the Forecasting Institute has primarily been to map inter-relations between the variables included in present educational statistics which are easy to observe and to measure (number of pupils, number of examinations, duration of study etc., by sex and faculty). In the long run it is desirable that the models be more strongly integrated with general social planning, i.e. number of teachers available and investment in new school buildings, and with those economic and social variables which might have a guiding effect on, for example, pupils' choice of education.

A short survey of the work on three models is given below.

I. The projection model of compulsory schooling

Sweden has only one compulsory school system, the courses and organisation of which has been decided upon by Parliament. A new compulsory school system, known as comprehensive schooling, comprising 9 years of education, is being gradually introduced between 1962 and 1972. This means that during the transitional period there will be two systems in operation. However, comprehensive schools already educate about 80 per cent of the pupils. The organisation of such schools is shown in diagram 1. During the first six years all instruction is common to all but a very small percentage of pupils, the handicapped and especially poorly gifted, who are instructed in special classes. During the 7th and 8th year in school, the classes are still kept together, but pupils are free to choose certain courses and subjects. During the 9th year, pupils are divided among nine different branches having a somewhat different orientation of subjects. Four branches are practical and vocational, and five are more theoretical.

The model for grades 1-6 of compulsory education takes as a starting point the fact that the age distribution by grade shows reliable stability. (Compulsory education starts the year in which a pupil is seven years old).

Table 1

Distribution by grade of pupils age 6-16

(in per cent)

	Age										
	6	7	8	9	10	11	12	13	14	15	16	(Total)
Grade 1	1.7	95.8	4.3	0.1								(101.9)
2		1.6	93.0	5.6	0.2							(100.4)
3			1.9	91.6	6.6	0.3						(100.4)
4				2.0	90.2	6.7	0.3					(99.2)
5					2.2	90.2	7.5	0.4				(100.3)
6						2.1	90.0	7.6	0.5			(100.2)
Total %	1.7	97.4	99.2	99.3	99.2	99.3						

From the distribution matrix (Table 1) the number of pupils in grades 1-6 can be computed with the help of population data. In grades 7-9 it is also important to study pupils' choice of subjects. The model is based on simple transitional probabilities such as : the probability that a pupil in grade 7 alternative B in year t, will be in grade 8 alternative C in year (t+1).

Pupils in grades 7-9 can choose from a large programme of subjects and courses. The choice to be made in the 7th year is between two different courses in English, two different courses in mathematics, 21 different courses in German, French, type-writing, handicrafts (major course) etc., and three different courses in music and handicrafts (minor course). The total number of combinations amounts to (2x2x21x3) = 252. The same system applies in the 8th and 9th years. If the transitional possibilities between all these different combinations were to be taken into consideration, a 1111x1111 matrix would be obtained. A model using a transitional matrix of this size just for primary education would be too troublesome. The different alternatives must therefore be aggregrated in a suitable way, and this aggregation should be made according to the problems of immediate interest. When planning school rooms or demand for teachers, pupils' choice of music and handicrafts can be of great interest, while their choices of these subjects could probably be completely ignored when studying recruitment to secondary schools. Because of this, the Forecasting Institute has attempted different levels of aggregation to obtain matrix dimensions such that they can be both run by the computer programme (see below) and are of a size allowing estimation of transition frequencies.

The above-sketched model for the comprehensive school has been programmed for computer (IBM 7090). In the computer programme the matrix of grades 7-9 (plus examinations after grade 9) cannot exceed the dimension 128 x 128. Each run of the programme can comprise only an even number of five years (with fixed coefficients in the matrix, but with changing data year by year for persons at each age).

No breakdown by sex is made in the model. As the proportions are more or less constant, the somewhat different educational patterns of female and male are of no practical importance for many possible applications of the model. In cases where a division by sex is required the model is applicable to each sex separately. In case a division by

Table 2

	Year t in grade									
	6	7 A	7 B	7 C	8 A	8 B	8 C	9 A	9 B	9 C
Year (t + 1) In grade 6										
7 A										
7 B										
7 C										
8 A				$P_{7C/8A}$						
8 B										
8 C										
9 A										
9 B										
9 C										

The transitional frequencies can be represented by, for example, $P_{7C/8A}$ which is the probability that one pupil in grade 7 alternative C year t, will be in grade 8 alternative A year (t + 1). The probability $P_{6/6}$ has no relevance in the model.

region is wanted, the model is applicable to each region separately.

Statistical data for deciding upon distribution coefficients by age of pupils in grades 1-6 are available (see table 1 above). So far, however, no statistical material has been collected which could be the basis for calculating the transitional probabilities for grades 7-9. The present statistics on pupils give only the annual distribution of all pupils. For practical use of the model, individual data are required on a large sample of pupils and their choices during at least two years. From this material (punched cards) the transitional coefficients could then easily be estimated with consideration of different aggregation alternatives. The interest shown by possible users of the model has so far been very moderate, and for that reason no measures have yet been taken to collect the necessary primary data for deciding upon the transitional coefficients.

Practical use of the model is further complicated by the fact that 20 per cent of the pupils in compulsory schools are still to be found in several other types of schools (the comprehensive school is being introduced successively in different regions; cf.page 95). Furthermore, the system of secondary education which is built on the compulsory school is at present being reorganised. It did not seem worth-while to construct models and to collect the necessary statistical flow data for those types of schools which are going to be liquidated within 2-4 years. As no statistics will be available for a long time on, for example, the new secondary school starting in 1966, one has had no reason as yet to make a formalized model for that part of the educational system. The best thing to do for the secondary school (i.e. the gymnasium) and for similar types of schools is probably to have a model with simple transitional probabilities of the same kind as are relevant for the 7th to the 9th years in the compulsory school. In the new secondary school some annual individual statistics will probably be collected.

The structure of the comprehensive school model is shown in diagrams 2 and 3.

II. The projection model of university level

Assumptions

The Swedish universities have faculties of theology, law, medicine, arts, social sciences and natural sciences. In addition to these faculties there are also colleges which provide education in Technology, Economics, Forestry, Agriculture, Pharmacy, Dentistry, etc. To enter these faculties students must, in general, have passed the Higher Certificate examination at the gymnasium. The gymnasium is divided into eight different main branches. Students from different branches show varying propensities in their choice of faculty or college.

The colleges and the faculty of medicine have restricted entry, a limit being fixed for the number of first year students every year. Competition among the candidates is as a rule very hard. The faculties of the universities, with the exception of the faculty of medicine, do not have restricted entry but accept all students who have passed the Higher Certificate examination. However, within these faculties there are certain subjects of study for which the number of students is limited. (The universities receive grant-in-aid automatically in order to be able to give all students a basic education .) As a result of the hard competition among students who want to enter the branches of higher education with restricted entry, many students start to study at the "free" (unrestricted as to number of new entrants each year) faculties while a waiting admittance to faculties with restricted entry.

As a rule the faculties with restricted entry have a well defined course of study leading to a degree after a fixed number of years. The faculties of arts, social sciences and natural sciences, on the other hand, cover a great number of independent subjects; here, as a rule, a university degree can be obtained after having studied at least two subjects during a period of at least three years. Many of the students, however, do not succeed in obtaining their degrees after only three years of study. Furthermore, many students study several subjects and do more course work than is formally required for a degree. Temporary interruptions in studies are also common. Finally, the definition of presence at a faculty is quite broad : every student who has taken part in at least one examination or lesson during a semester is registered. All students at higher levels (post-graduates) are included.

The conditions for the construction of the comprehensive school model and the university model were different. The comprehensive school model was constructed as an experiment without any prior contact with possible users, and with limited consideration taken of available statistical information. The university model was constructed to make possible a projection of the number of students 15 years ahead. Therefore, available statistical information had to be used. This consisted of :

(i) Individual data for registered students during a period of 10 years by branch in the gymnasium, year of examination from the gymnasium, and earlier enrolment, if any, at university level ;

(ii) Individual data on the number of students present at the universities in 1962-63 by year of enrolment.

However, data was not available on how many of those who had left university had graduated or not. Data was available on the number of examinations in different years, but there was no information on the initial enrolment year of the graduates or on the year they left university. This is especially important for post-graduate students who usually leave university many years after graduation. Within a short time an individual register, covering all the students at university level, will be available on tape.

The construction of the model

Due to the existence of faculties with restricted entry, it has not been possible to use a simple model with transition probabilities for further studies after secondary school at different faculties and colleges. Instead the model is built as follows :

(i) Total admittance to universities and colleges (first enrolment) by sex, branch of gymnasium and number of waiting-years (years between examination from gymnasium and first enrolment at university or college; 0-3 years have been taken into consideration);

(ii) Inflow of students to faculties with restricted entry (first enrolment) by sex, branch of gymnasium and waiting-years;

(iii) Total admittance to universities and colleges (see (i)) less number of students admitted to faculties with restricted entry (see (ii)). The rest of the students are distributed among faculties without restricted entry.

(iv) Starting with the number of students enrolled during the most recent years (the length of study, and hence the number of years spent by students, varies between different faculties and colleges) the number of students present is computed for the different faculties and colleges.

According to the above sketched method it has been assumed, when making the computations, that students with more than three waiting years form a constant part of total admittance to each faculty and college, and that students who are present at each faculty and college for an extemely long period (post-graduates and slow students) form a constant percentage of the total number of students present. A sketch of the model is shown in Appendix 1 . The formalized model is shown in Appendix 2.

As pointed out earlier, this model was constructed to make projections of the number of students at universities possible. The model gives data about the inflow to faculties as well as about the number of those leaving the faculties. As soon as adequate statistics are available the model will be supplemented with assumptions about the percentage of students who leave universities with a degree. Once we have had some experience with the model we shall take into consideration whether the model could, in some respects, be less detailed. Thus far, computation has been carried out on the model only once, and that manually. At the moment, a new count is being prepared in which a series of standardised forms will be used for assumptions and for results. After this second manual count the model might be prepared for EDP.

III. Teachers in primary schools, secondary schools, etc.

As in many other countries there is a serious shortage of teachers in Sweden. For this reason it is important to construct a model for the demand and supply of school teachers. This model can be regarded as a further step in developing the models of the above mentioned schools. As long as there are no useful models for pupils in these schools, the model of demand for teachers must be built upon schematic hypotheses regarding the number of pupils and the size of classes in the schools in question. There are rather good statistics on the supply of teachers, due to the fact that most Swedish teachers are already included in the comprehensive teacher-statistics collected on an individual basis. At the moment a national teacher-register is being constructed on tape.

One of the difficulties in constructing a model for demand and supply of teachers is that we have a demand for teachers giving lessons in specific subjects, while teachers normally instruct in several subjects. Teachers sometimes also teach subjects for which they are not formally qualified. Due to the fact that many teach at several types of schools, it is necessary that the model, in order to be realistic, should cover a very large part of the educational system, including vocational schools and adult education.

Some general comments

(i) Model building is hampered by the shortage of suitable statistical data, and by the fact that information on the structure of the educational system necessary for model building is to some extent missing. The work on building the model has, for this reason, been combined with the collecting of ad hoc data and the planning of new regular educational statistics (often in connection with the introduction of administrative data systems). Furthermore, the work on the educational models has been combined with a qualitative mapping of the structure of the educational system.

(ii) The objective of the work on the educational models should be such that these models become tools in administrative educational planning. However, it has been difficult for administrators to explain exactly what they require from the models. The demands put forward have in some cases been laid out in an unrealistical manner, or have been impossible to carry out due to the present shortage of statistical data.

(iii) Opinions about the possibilities of using the models for long-term (5-15 years) projections are divided. For this reason, the effects of variations in the different variables and coefficients of the model need to be analysed. Alternatively, completely different models should be used for short-run and long-run forecasts. It seems possible to construct a computer programme in such a way that interpolation can automatically be performed on a matrix or a vector in the computer, thus easing the introduction of linear trends for the coefficients of the model.

(iv) It is important to try to introduce social, economic and other variables, to enable the models to predict changes of the transitional probabilities which are now treated as constants.

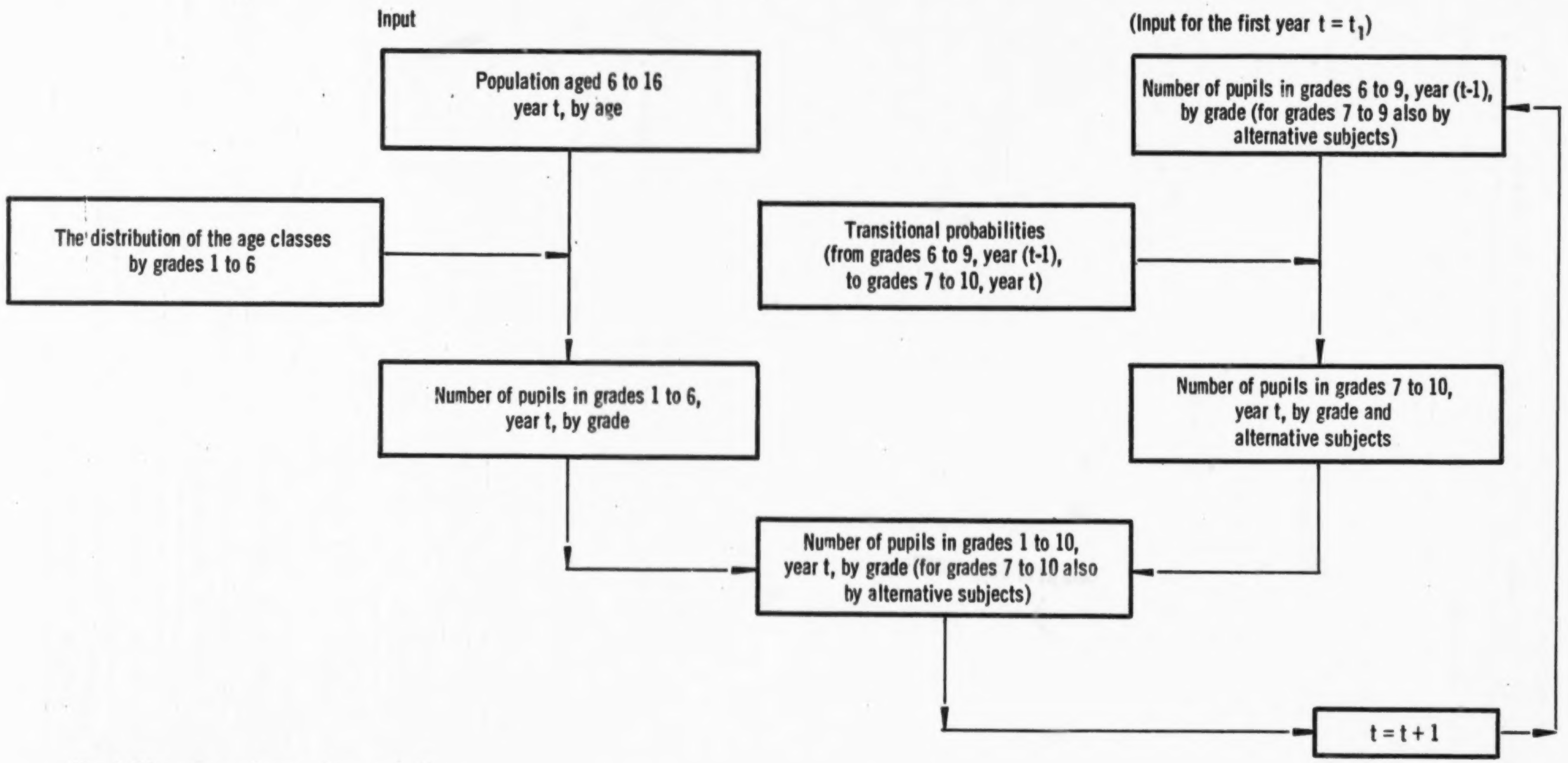
APPENDIX I

Diagram 1: The Compulsory school

The pupils are kept together in the same classes from grade 1 to grade 8. In grade 9 the pupils are divided into 9 different streams, mostly in separate classes.

	5 different streams	4 different streams
Grade 9	4 weekly periods: 11 optional subjects (alternative courses or couples of subjects) e.g. German, French,	22 weekly periods: 5 optional groups of practical or vocational subjects (typewriting, engineering etc.)
	3 weekly periods: English, general or modified course	
	4 weekly periods: mathematics, general or modified course	
	4 weekly periods: 2 of the subjects: music, art, textile work or wood and metal work	2 weekly periods: 1 of the subjects: English, music or art
	20 weekly periods common to all pupils	11 weekly periods common to all pupils
Grade 8	4 weekly periods: 18 optional subjects (alternative courses or couples of subjects)e.g. German, French, typewriting, mercantile course	7 weekly periods: 7 optional subjects or groups of subjects, e.g. typewriting, technical work
	3 weekly periods: English, general or modified course	
	4 weekly periods: mathematics, general or modified course	
	4 weekly periods: 2 of the subjects: music, art, textile work or wood and metal work	
	20 weekly periods common to all pupils	
Grade 7	5 weekly periods: 21 optional subjects (alternative courses or groups of subjects) e.g. German, French, mathematics (extra course), typewriting	
	4 weekly periods: English, general or modified course	
	4 weekly periods: mathematics, general or modified course	
	2 weekly periods: 1 of the subjects music, textile work or wood and metal work	
	20 weekly periods common to all pupils	
Grade 6	32 weekly periods common to all pupils	
Grade 5	32 weekly periods common to all pupils	
Grade 4	30 weekly periods common to all pupils	
Grade 3	25 weekly periods common to all pupils	
Grade 2	20 weekly periods common to all pupils	
Grade 1	19 weekly periods common to all pupils	

Diagram 2. THE PROJECTION MODEL OF THE COMPULSORY SCHOOL

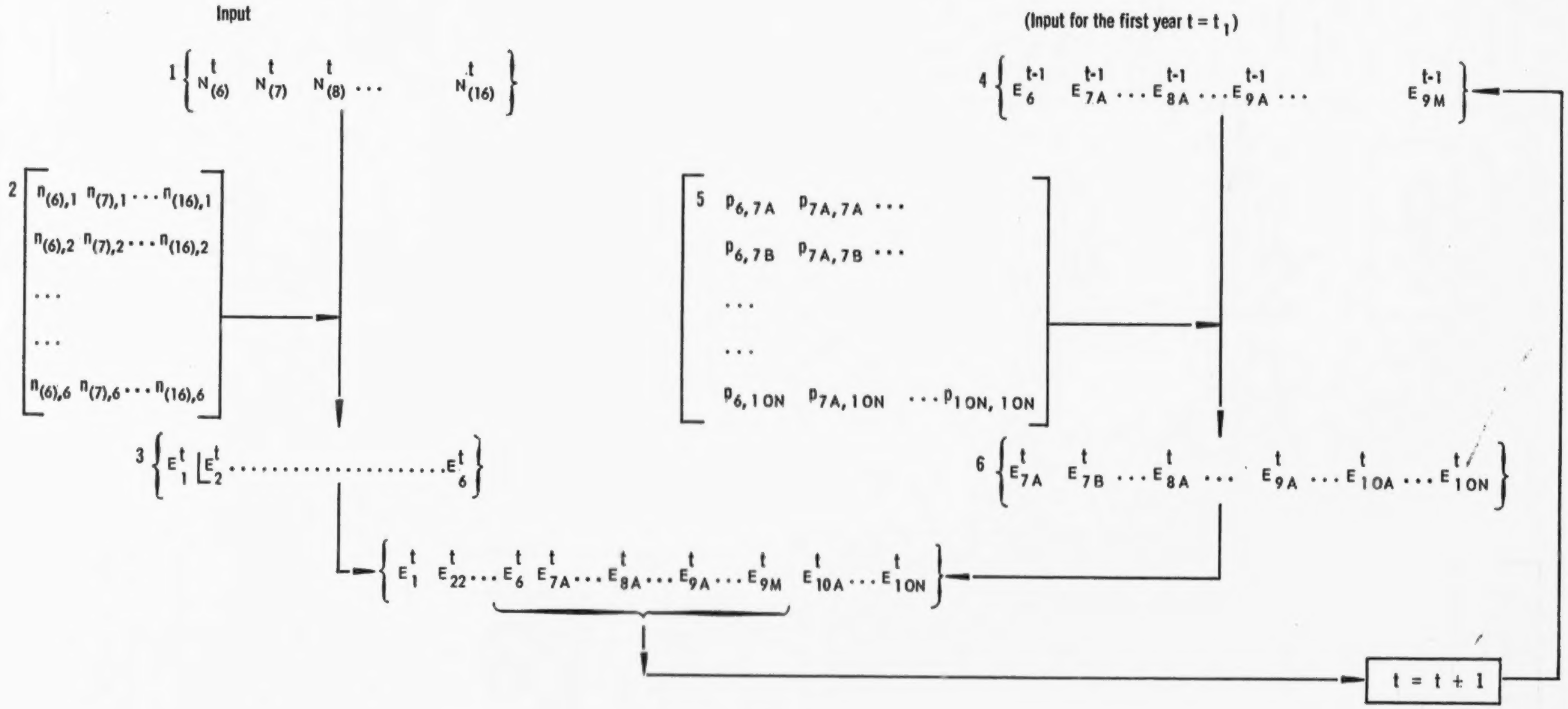

Input
(Input for the first year t = t_1)
Population aged 6 to 16 year t, by age
Number of pupils in grades 6 to 9, year (t-1), by grade (for grades 7 to 9 also by alternative subjects)
The distribution of the age classes by grades 1 to 6
Transitional probabilities (from grades 6 to 9, year (t-1), to grades 7 to 10, year t)
Number of pupils in grades 1 to 6, year t, by grade
Number of pupils in grades 7 to 10, year t, by grade and alternative subjects
Number of pupils in grades 1 to 10, year t, by grade (for grades 7 to 10 also by alternative subjects)
t = t + 1

(Grade 10 means graduation from grade 9)

Input : Population aged 6 to 16 for every year t (box 1);
and the number of pupils in grades 6 to 9, year (t_1 - 1),
by grade and alternative courses (box 5, t_1 = first year
of the projection)

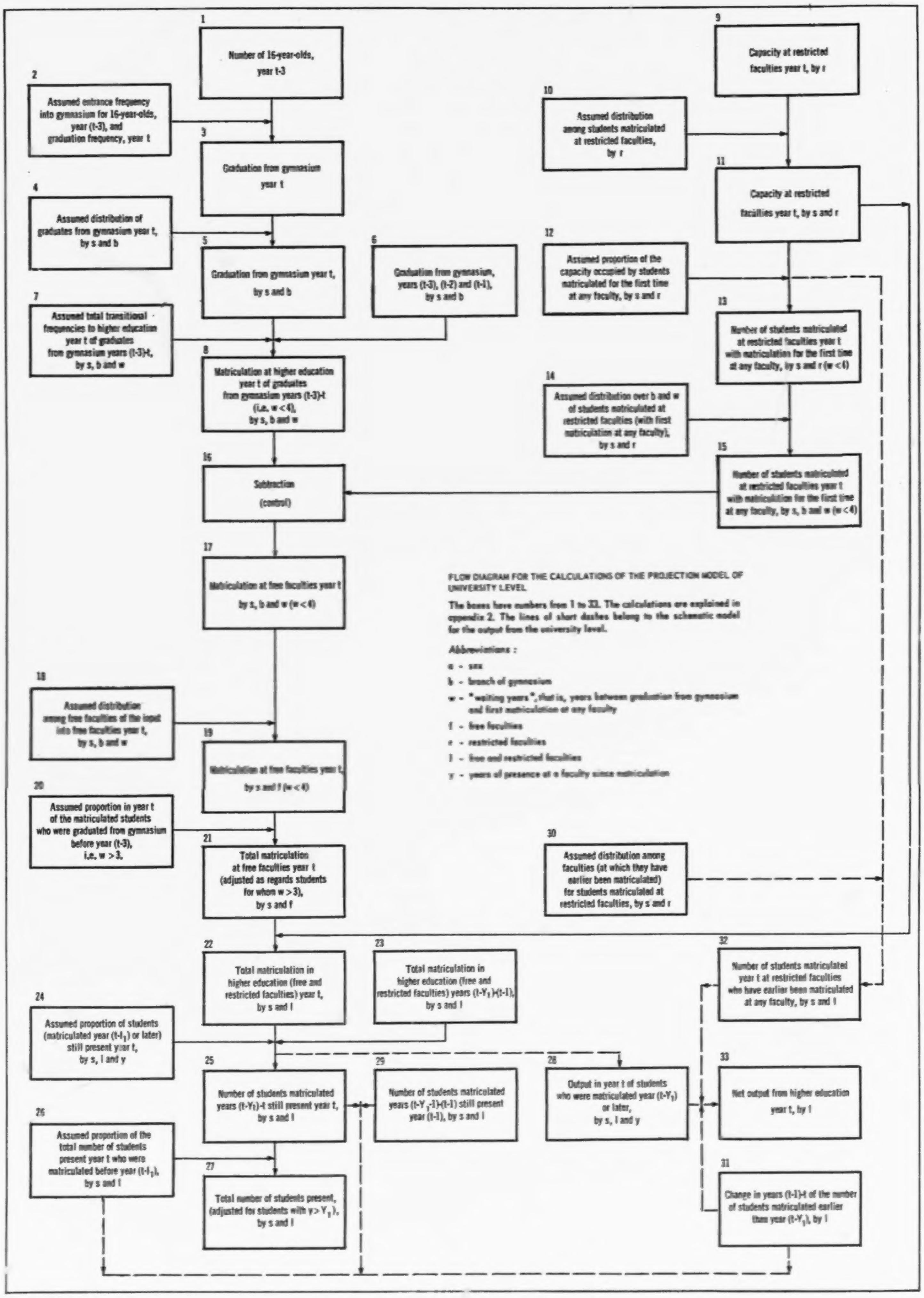
Output : Number of pupils in grades 1 to 10 by grade (for grades 7
to 10 also by alternative subjects). (Box 4).

Diagram 3. THE PROJECTION MODEL OF THE COMPULSORY SCHOOL


Input
(Input for the first year $t = t_1$)
1 $\left\{ N^t_{(6)} \; N^t_{(7)} \; N^t_{(8)} \cdots N^t_{(16)} \right\}$
2 $\begin{bmatrix} n_{(6),1} & n_{(7),1} \cdots n_{(16),1} \\ n_{(6),2} & n_{(7),2} \cdots n_{(16),2} \\ \cdots \\ \cdots \\ n_{(6),6} & n_{(7),6} \cdots n_{(16),6} \end{bmatrix}$
3 $\left\{ E^t_1 \; E^t_2 \cdots\cdots E^t_6 \right\}$
4 $\left\{ E^{t-1}_6 \; E^{t-1}_{7A} \cdots E^{t-1}_{8A} \cdots E^{t-1}_{9A} \cdots E^{t-1}_{9M} \right\}$
5 $\begin{bmatrix} p_{6,7A} & p_{7A,7A} \cdots \\ p_{6,7B} & p_{7A,7B} \cdots \\ \cdots \\ \cdots \\ p_{6,10N} & p_{7A,10N} \cdots p_{10N,10N} \end{bmatrix}$
6 $\left\{ E^t_{7A} \; E^t_{7B} \cdots E^t_{8A} \cdots E^t_{9A} \cdots E^t_{10A} \cdots E^t_{10N} \right\}$
$\left\{ E^t_1 \; E^t_{22} \cdots E^t_6 \; E^t_{7A} \cdots E^t_{8A} \cdots E^t_{9A} \cdots E^t_{9M} \; E^t_{10A} \cdots E^t_{10N} \right\}$
$t = t + 1$

For an explanation of the formalized model see diagram 2.

The numbers of the vectors and the matrixes correspond to the numbers of the boxes in diagram 2. Grade 10 means the output from grade 9.


1
Number of 16-year-olds, year t-3
2
Assumed entrance frequency into gymnasium for 16-year-olds, year (t-3), and graduation frequency, year t
3
Graduation from gymnasium year t
4
Assumed distribution of graduates from gymnasium year t, by s and b
5
Graduation from gymnasium year t, by s and b
6
Graduation from gymnasium, years (t-3), (t-2) and (t-1), by s and b
7
Assumed total transitional frequencies to higher education year t of graduates from gymnasium years (t-3)-t, by s, b and w
8
Matriculation at higher education year t of graduates from gymnasium years (t-3)-t (i.e. w < 4), by s, b and w
9
Capacity at restricted faculties year t, by r
10
Assumed distribution among students matriculated at restricted faculties, by r
11
Capacity at restricted faculties year t, by s and r
12
Assumed proportion of the capacity occupied by students matriculated for the first time at any faculty, by s and r
13
Number of students matriculated at restricted faculties year t with matriculation for the first time at any faculty, by s and r (w < 4)
14
Assumed distribution over b and w of students matriculated at restricted faculties (with first matriculation at any faculty), by s and r
15
Number of students matriculated at restricted faculties year t with matriculation for the first time at any faculty, by s, b and w (w < 4)
16
Subtraction (control)
17
Matriculation at free faculties year t by s, b and w (w < 4)
18
Assumed distribution among free faculties of the input into free faculties year t, by s, b and w
19
Matriculation at free faculties year t, by s and f (w < 4)
20
Assumed proportion in year t of the matriculated students who were graduated from gymnasium before year (t-3), i.e. w > 3.
21
Total matriculation at free faculties year t (adjusted as regards students for whom w > 3), by s and f
22
Total matriculation in higher education (free and restricted faculties) year t, by s and l
23
Total matriculation in higher education (free and restricted faculties) years (t-Y_1)-(t-1), by s and l
24
Assumed proportion of students (matriculated year (t-I_1) or later) still present year t, by s, l and y
25
Number of students matriculated years (t-Y_1)-t still present year t, by s and l
26
Assumed proportion of the total number of students present year t who were matriculated before year (t-I_1), by s and l
27
Total number of students present, (adjusted for students with y > Y_1), by s and l
28
Output in year t of students who were matriculated year (t-Y_1) or later, by s, l and y
29
Number of students matriculated years (t-Y_1-1)-(t-1) still present year (t-1), by s and l
30
Assumed distribution among faculties (at which they have earlier been matriculated) for students matriculated at restricted faculties, by s and r
31
Change in years (t-1)-t of the number of students matriculated earlier than year (t-Y_1), by l
32
Number of students matriculated year t at restricted faculties who have earlier been matriculated at any faculty, by s and l
33
Net output from higher education year t, by l

FLOW DIAGRAM FOR THE CALCULATIONS OF THE PROJECTION MODEL OF UNIVERSITY LEVEL

The boxes have numbers from 1 to 33. The calculations are explained in appendix 2. The lines of short dashes belong to the schematic model for the output from the university level.

Abbreviations :

s - sex
b - branch of gymnasium
w - "waiting years", that is, years between graduation from gymnasium and first matriculation at any faculty
f - free faculties
r - restricted faculties
l - free and restricted faculties
y - years of presence at a faculty since matriculation

APPENDIX 2

The formalized projection model of university level

The description of the model follows the flow diagram in appendix 1. The numbers in the left margin correspond to the numbers of the boxes in the flow diagram.

The different quantities have abbreviations, starting with N if it is a number or P if it is a probability or frequency. The rest of the abbreviations correspond to key words, which are underlined in the text. The quantities are mostly distributed on some of the following categories.

$s = 1, 2 = S$	sex
$b = 1, 2, .. B$	branch of gymnasium (up to now $B = 7$ for the present gymnasium, and $B = 5$ for the "new" gymnasium starting 1966)
$w = 0, 1, .. W$	waiting years, that is, years between graduation from gymnasium and first matriculation at any faculty or college. In the model it is supposed that all freshmen at the free faculties have not previously been matriculated at any faculty (which is an almost true assumption). It is supposed that freshmen with more than 3 waiting years constitute a small, more or less constant, proportion of the total number of freshmen.
$f = 1, 2, .. F$	free faculties at the universities. There are at the moment 5 different faculties without restricted entry (theology, law, arts, social sciences and natural sciences). The earlier faculty of arts and social sciences was split up as late as 1964. As

	we lack sufficient data from the two new faculties we still handle them as one faculty in the model ($F = 4$).
$r = 1, 2, ..R$	restricted faculties. This group consists of the faculties of medicine and economics, but also of colleges and professional schools which provide education in technology, forestry, agriculture, pharmacy, dentistry etc. Up to now $R = 12$.
$l = 1, 2, ..L$	free and restricted faculties. Up to now $L = F + R = 4 + 12 = 16$
$y = 0,1,2, ..Y_l$	years of presence at a faculty since matriculation. Due to drop-outs, variations in the individual time needed for graduation and other factors, the remaining portion of students who matriculated in a certain year gradually decreases. The rate of decrease differs between the faculties due to (among others) differences in the ideal time for graduation. For every faculty, Y_l has been chosen so that the portion of students still present is less than 15 %. Students staying for a longer time (for example post-graduate students) are supposed to be a more or less constant proportion of the whole stock of students present. Up to now $2 \leq Y_l \leq 9$.

The model is based on a great many assumptions of constancy which are not yet fully mapped and are not mentioned in this short description of the model.

GRADUATION FROM GYMNASIUM

1. Number of 16-year-olds, year (t-3)

   $N16^{t-3}$

2. Assumed entrance frequency into gymnasium for 16-year-olds year (t-3), and the graduation frequency year t of those who entered the gymnasium year (t-3)

   Peg^{t-3} $\quad$ Pgf^{t}

3. Graduation from gymnasium year t

   $Ngg^{t} = N16^{t-3} \times Peg^{t-3} \times Pgf^{t}$

4. Assumed distribution of graduates from gymnasium year t, by sex and branch of gymnasium

   $Pgg^{t}_{(s,b)}$

   s = 1, 2 sex

   b = 1, 2... branch of gymnasium

   $$\sum_{s}^{S} \sum_{b}^{B} Pgg^{t}_{(s,b)} = 1.00$$

5. Graduation from gymnasium year t, by sex and branch of gymnasium

   $Ngg^{t}_{(s,b)} = Ngg^{t} \times Pgg^{t}_{(s,b)}$

   s = 1, 2 sex

   b = 1, 2... branch of gymnasium

6. Graduation from gymnasium year (t-3), (t-2) and (t-1) by sex and branch of gymnasium

$Ngg^{t-3}_{(s,b)}$, $Ngg^{t-2}_{(s,b)}$, $Ngg^{t-1}_{(s,b)}$

s = 1, 2 sex

b = 1, 2... branch of gymnasium

TOTAL MATRICULATION IN HIGHER EDUCATION

7. Assumed total transitional frequences to higher education year t of graduates from gymnasium years (t-3) to t (only the first matriculation for every student at any faculty), by sex, branch of gymnasium and waiting years

$Ptt^{t-w}_{(s,b,w)}$

s = 1, 2 sex

b = 1, 2... branch of gymnasium

w = 0,1,2,3 waiting years

$$\sum_{\omega}^{W} Ptt^{t}{}_{(s,b,w)} \leqslant 1.00$$

8. Matriculation in higher education (free and restricted faculties) year t of graduates from gymnasium years (t-3) to t, by sex, branch of gymnasium and waiting years (only students with at most 3 waiting years and who have not before been matriculated at any faculty)

$$Nmhe^{t}_{(s,b,w)} = Ngg^{t-w}_{(s,b)} \times Ptt^{t-w}_{(s,b,w)}$$

s = 1, 2 sex

b = 1, 2... branch of gymnasium

w = 0,1,2,3 waiting years

MATRICULATION AT RESTRICTED FACULTIES

9. Capacity at restricted faculties year t, by faculty

$Ncr^{t}_{(r)}$

r = 1, 2... restricted faculties

10. Assumed sex distribution among students matriculated at restricted faculties year t, by faculty

$Ps^{t}_{(s,r)}$

s = 1, 2 sex

r = 1, 2... restricted faculties

$$\sum_{s}^{2} Ps^{t}_{(s,r)} = 1.00$$

11. Capacity at restricted faculties year t, by sex and faculty

$$Ncr^{t}_{(s,r)} = Ncr^{t}_{(r)} \times Ps^{t}_{(s,r)}$$

s = 1, 2 sex

r = 1, 2... restricted faculties

12. Assumed proportions of the capacity at restricted faculties year t occupied by students matriculated for the first time at any faculty and with at most 3 waiting years, by sex and faculty

$Prft^{t}_{(s,r)}$

s = 1, 2 sex
r = 1, 2... restricted faculties

13. Number of students matriculated at restricted faculties year t with matriculation for the first time at any faculty and with at most 3 waiting years, by sex and faculty

$$Nrft^{t}_{(s,r)} = Ncr^{t}_{(s,r)} \times Prft^{t}_{(s,r)}$$

s = 1, 2 sex
r = 1, 2... restricted faculties

14. Assumed distribution year t over branch of gymnasium and waiting year of students matriculated at restricted faculties (with first matriculation at any faculty and with at most 3 waiting years), by sex and faculty

$Pbw^{t}_{(b,w/s,r)}$

s = 1, 2 sex
r = 1, 2... restricted faculties
b = 1, 2... branch of gymnasium
w = 0,1,2,3 waiting years

$$\sum_{b,w}^{B,W} Pbw^{t}_{(b,w/s,r)} = 1.00$$

15. Number of students matriculated at restricted faculties year t with matriculation for the first time at any faculty and with at most 3 waiting years, by sex, branch of gymnasium and waiting years

$$Nrft^{t}_{(s,b,w)} = \sum_{r}^{R} \left[Nft^{t}_{(s,r)} \times Pbw^{t}_{(b,w/s,r)} \right]$$

s = 1, 2 sex

r = 1, 2..R restricted faculties

b = 1, 2... branch of gymnasium

w = 0,1,2,3 waiting years

MATRICULATION AT FREE FACULTIES

16. Subtraction
(If the subtraction gives any negative number of students the assumptions in the boxes 10, 12 and 14 will have to be revised.)

17. Matriculation at free faculties year t, by sex, branch of gymnasium and waiting years (only students with at most 3 waiting years and who have not before been matriculated at any faculty)

$$Nf^{t}_{(s,b,w)} = Nmhe^{t}_{(s,b,w)} - Nrft^{t}_{(s,b,w)}$$

s = 1, 2 sex

b = 1, 2... branch of gymnasium

w = 0,1,2,3 waiting year

18. Assumed distribution among free faculties of the input into free faculties year t, by sex, branch of gymnasium and waiting years (for students with at most 3 waiting years and who have not before been matriculated at any faculty)

$Pf^{t}_{(f/s,b,w)}$

f = 1, 2... free faculties

s = 1, 2 sex

b = 1, 2... branch of gymnasium

w = 0,1,2,3 waiting years

$$\sum_{f}^{F} Pf^{t}_{(f/s,b,w)} = 1.00$$

19. Matriculation at free faculties year t, by sex and free faculty (only students with at most 3 waiting years and who have not before been matriculated at any faculty)

$$Nf^{t}_{(s,f)} = \sum_{b,w}^{B,W} \left[Nf^{t}_{(s,b,w)} \times Pf^{t}_{(f/s,b,w)} \right]$$

f = 1, 2... free faculties

s = 1, 2 sex

b = 1, 2... branch of gymnasium

w = 0,1,2,3 waiting years

20. Assumed proportion year t of the matriculated students at each faculty who were graduated from gymnasium before year (t-3) (more than 3 waiting years, i.e. $\underline{w > 3}$) or who have previously been matriculated at any faculty

$P(w > 3)^{t}_{(f)}$

f = 1, 2... free faculties

21. Total matriculation at free faculties year t, by sex and free faculty (adjusted as regards matriculated students with more than 3 waiting years or who have previously been matriculated at any faculty)

$$Nfa^{t}_{(s,f)} = Nf^{t}_{(s,f)} \times \frac{1}{1 - P(w > 3)^{t}_{(f)}}$$

s = 1, 2 sex

f = 1, 2... free faculties

STUDENTS IN HIGHER EDUCATION

22. Total matriculation in higher education (free and restricted faculties) year t, by sex and faculty

$$Ntmhe^{t}_{(s,l)} = Nfa^{t}_{(s,f)} + Nr^{t}_{(s,r)}$$

s = 1, 2 sex

f = 1, 2... free faculties

r = 1, 2... restricted faculties

l = 1, 2... free and restricted faculties (L = F + R)

23. Total matriculation in higher education (free and restricted faculties) year $(t-Y_1)$ to $(t-1)$, by sex and faculty

$$Ntmhe^{t-1}_{(s,l)}, \quad Ntmhe^{t-2}_{(s,l)}, \quad \text{etc.} \quad Ntmhe^{t-Y_1}_{(s,l)}$$

$s = 1, 2$ sex

$l = 1, 2...$ free and restricted faculties

$Y_l =$ maximum number of years since matriculation taken into consideration for faculty ℓ

24. Assumed proportion of students (matriculated year $(t-Y_l)$ or later) still present year t, by sex, faculty and years since matriculation

$$Pp^{t-y}_{(s,l,y)}$$

$s = 1, 2$ sex

$l = 1, 2...$ (free and restricted) faculties

$y = 0,1..Y_l$ years since matriculation (Y_l differs between the faculties)

25. Number of students matriculated years $(t-Y_l)$ to t still present year t, by sex and faculty

$$Np^{t}_{(s,l)} = \sum_{y}^{Y_l} \left[Ntm^{t-y}_{(s,l)} \times Pp^{t-y}_{(s,l,y)} \right]$$

$s = 1, 2$ sex

$l = 1, 2...$ (free and restricted) faculties

$y = 0,1,2..Y_l$ years since matriculation (Y_l differs between the faculties)

26. Assumed proportion of the total number of students present year t who were matriculated before year $(t-Y_l)$, by sex and faculty (i.e. $\underline{y > Y_l}$)

$$P(y > Y_l)^{t}_{(s,l)}$$

$s = 1, 2$ sex

$l = 1, 2...$ (free and restricted) faculties

$Y_l =$ maximum number of years since matriculation taken into consideration for faculty ℓ

27. Total number of students *present* year t, by sex and faculty (*adjusted* for the proportion of students who were matriculated before year $(t-Y_l)$

$$Npa^{t}_{(s,l)} = Np^{t}_{(s,l)} \times \frac{1}{1 - P(y > Y_l)^{t}_{(s,l)}}$$

$s = 1, 2$ sex

$l = 1, 2...$ (free and restricted) faculties

$Y_l =$ maximum number of years since matriculation taken into consideration for faculty ℓ

OUTPUT FROM HIGHER EDUCATION

(a very rough model which gives no information about the number of students who have graduated from universities and professional schools)

28. *Output* year t of students who have been matriculated year $(t-Y_l)$ or later, by sex, faculty and years since matriculation

$$Nop^{t}_{(s,l,y)} = Ntm^{t-y}_{(s,l)} \times \left[Pp^{t-y}_{(s,l,y)} - Pp^{t-y}_{(s,l,y+1)} \right]$$

$s = 1, 2$ sex

$l = 1, 2...$ (free and restricted) faculties

$y = 0,1,2..Y_l$ years since matriculation (Y_l differs between the faculties)

$$Pp^{t}_{(s,l,y)} = 0 \text{ in the model when } y > Y_l$$

$$Nop^{t}_{(s,l)} = \sum_{y}^{Y_l} Nop^{t}_{(s,l,y)}$$

29. Number of students matriculated years $(t-Y_l-1)$ to $(t-1)$ still present year $(t-1)$, by sex and faculty

$$Np^{t-1}_{(s,l)}$$

$s = 1, 2$ sex

$l = 1, 2\ldots$ (free and restricted) faculties

$Y_l =$ maximum number of years since matriculation taken into consideration faculty ℓ

30. Assumed distribution among faculties (at which they have earlier been matriculated) of students matriculated year t at restricted faculties, by sex and restricted faculty.

$$Pem^{t}_{(s,r,l)}$$

$s = 1, 2$ sex

$r = 1, 2\ldots$ restricted faculties (at which the students matriculate year t)

$l = 1, 2\ldots$ (free and restricted) faculties (at which the students have earlier been matriculated)

31. Change in years (t-1) to t of the number of students matriculated earlier than year $(t-Y_l)$, by faculty

$$Nch^{t}_{(l)} = \sum_{s}^{S} Nch^{t}_{(s,l)} =$$

$$= \sum_{s}^{S} \left[P(y > Y_1)_{(s,l)}^{t} \times Npa_{(s,l)}^{t} - P(y > Y_1)_{(s,l)}^{t-1} \times Npa_{(s,l)}^{t-1} \right]$$

$s = 1, 2$ sex

$l = 1, 2...$ (free and restricted) faculties

(The proportion of post-graduate students is among other things a function of the number of freshmen some years earlier. Many post-graduate students have jobs as instructors, lecturers etc. The number of post-graduates is to some extent also a function of the number of undergraduate students ("feed-back"). As long as the exact mechanism is unknown, the above calculations may give a rough estimate of the variation of students matriculated before year $(t-Y_1)$. The rough feed-back model does not justify a division by sex.)

32. Number of students matriculated year t at restricted faculties who have earlier been matriculated at any faculty (or with more than 3 waiting years), by sex and faculty. The students who have earlier been matriculated at any faculty have left these faculties in the way described in paragraph 28.

$$Nem_{(s,l)}^{t} = \sum_{r}^{R} \left[Pem_{(s,r,l)}^{t} \times (Ncr_{(s,r)}^{t} - Nrft_{(s,r)}^{t}) \right]$$

$s = 1, 2$ sex

$r = 1, 2...$ restricted faculties

33. Net output from higher education year t, by (free or restricted) faculties

$$Nnop_{(l)}^{t} = \sum_{s}^{S} \left[Nop_{(s,l)}^{t} - Nem_{(s,l)}^{t} - Nch_{(s,l)}^{t} \right]$$

$s = 1, 2$ sex

$l = 1, 2...$ (free and restricted) faculties

A MATHEMATICAL MODEL OF THE NORWEGIAN EDUCATIONAL SYSTEM

by Tore Thonstad

Introduction

In order to gain insight into the working of an educational system, it is useful to study the long-run implications of present educational propensities. For instance, what are the implications for the educational composition of the labour force ? What fraction of today's pupils in a certain type of school will eventually graduate from a university? And so on.

In many analyses of educational systems, in particular in making forecasts of enrolment, graduation, etc., a wide range of ratios is used. It is assumed for instance that a given percentage of the pupils enrolled in a school will pass their exams successfully, and that a given fraction of them will go on to another school, etc. Such ratios are, of course, dependent upon several factors : school capacities and admission policies, the intellectual abilities of the pupils, availability of scholarships, the quality of teaching, etc. In this paper we will take a great number of ratios of the above-mentioned type as given, and compute the output of graduates and various characteristics of the system on this basis. The analysis can be looked upon as a formalization, systematization and generalization of the ad hoc procedures used by many practical forecasters.

The main advantage of the approach of the present paper is that it provides an overall model of the entire school system of a country. The approach makes it possible to give forecasts of school attendance in all parts of the school system, as well as final graduation from all the different types of schools, in one operation, thus achieving consistency between the different forecasts.

It should be stressed that this paper is a progress report about parts of my work in this field, and that a much more comprehensive final report will come later.

The paper is arranged as follows: first, a multi-activity model of the Markov chain type of the educational system is presented (Section I). Some mathematical implications of the model are given in Section II. The adaptation of the model for empirical application is discussed in Section III. The data used are briefly described in Section IV. Some examples of derived characteristics of the Norwegian educational system are given in Section V.

I. The Markov chain model

Some previous authors have used mathematical models to represent the structure of an educational system.[1] In some cases, the school system has been conceived of as a ladder, representing the school stages in succession, no attempts being made to study simultaneously the branching in educational specialisations. In a few cases, more general models have been constructed (see in particular [1] , [6] and [7]). Our own task will be to develop a multi-activity model of an educational system and apply it to Norwegian data.

---

[1] See references to Brown and Savage [1] , Correa, [2] , Gani [4] , Moser and Redfern [6] and Stone [7] .

The theory of absorbing Markov chains provides a useful framework for our analysis; and before formulating our model, we shall very briefly state the main assumptions of one type of absorbing Markov chains. [1]

We will first give a definition of a discrete Markov chain. Suppose that we are investigating a physical system which can exist only in one of a finite number of states, and which can change its state only at discrete points of time. As we shall soon see, a state can for instance be a certain school. Let the states be designated by the integers $1,2,\dots,N$, and the times by $0,1,2,\dots$. We now assume that there is a fixed probability, c_{ij}, that a system in state i at time t will transform into state j at time $t+1$. In our application this may correspond to a probability for transferring from one school to another. Such probabilities are defined for all (i,j), and are presented in a $N \times N$ transition matrix :

$$(1) \qquad C = \begin{bmatrix} c_{11} & \cdots & c_{1N} \\ \vdots & & \vdots \\ c_{N1} & \cdots & c_{NN} \end{bmatrix}$$

In the simplest case, C is assumed to be independent of time. Because the elements are probabilities, all c_{ij} are non-negative, and the row totals equal unity. The latter condition means that a "particle" (in our application: a pupil) in state i at time t must be in one of the N permissible states at time $t+1$.

A special type of Markov chain, called an absorbing chain, will be used here. A state in a Markov chain is an absorbing state if there is a zero probability of leaving it. A Markov chain is absorbing if it has at least one absorbing state, and from every state it is possible to go to an absorbing state (although not necessarily in one step).

If there are E absorbing and S non-absorbing states, the transition matrix will have the following form (when we put the absorbing states first):

---

[1] Our brief exposition is largely based on J.C. Kemeny, H. Mirkil, J.L. Snell and G.L. Thompson [5] . See also W. Feller [3].

$$C = \left[\begin{array}{c|c} I_E & 0 \\ \hline R & Q \end{array}\right] \qquad (2)$$

I_E is an E x E identity matrix, 0 is an E x S zero matrix, R is an S x E matrix, and Q is an S x S matrix. R gives the probabilities for transitions from non-absorbing to absorbing states, and Q gives the probabilities for transfers between non-absorbing states.

We now assume that an individual is either in one of the S school activities of one year's duration, or in one of the E absorbing states. These absorbing states will be interpreted as different completed educations. We will assume that any final drop-out from the school system can be classified into one of the E education categories (in our empirical approach we shall treat mortality during schooling as absorption into one of the E categories).

In our application we will deviate from the approach briefly outlined above by using a deterministic (non-stochastic) instead of a stochastic approach. This means that we will interpret the elements, c_{ij}, of (1) as fixed transition ratios, not as probabilities. For the individuals in any of the E + S states (for instance state i), there is set of transition ratios (c_{ij}) representing the fractions going to each of the states (schools or finished educations) next year.

We define the following non-negative transition ratios:

(3) q_{hk} = fraction of pupils in school activity h in a given year who will next year be in school activity k ($h,k=1,\ldots,S$).

(4) r_{he} = fraction of pupils in school activity h in a given year who will leave school at the end of the year with final education e ($h=1,\ldots,S$; $e=1,\ldots,E$).

These ratios are arranged in the following two transition matrices (cf. (2)):

$$(5) \qquad Q = \begin{bmatrix} q_{11} & \cdots\cdots & q_{1S} \\ \vdots & & \vdots \\ q_{S1} & \cdots\cdots & q_{SS} \end{bmatrix}$$

$$(6) \qquad R = \begin{bmatrix} r_{11} & \cdots\cdots & r_{1E} \\ \vdots & & \vdots \\ r_{S1} & & r_{SE} \end{bmatrix}$$

The diagonal elements of Q are repetition ratios, that is, the fraction of pupils in a certain school activity who will be in the same activity next year. Our assumptions imply that infinite repetition is possible. However, the error introduced through this assumption is probably very small.

There will be a tendency for persons getting a given final education to come from the same final school activity. If that were always the case, there would be only one element in each column of R. However, one reason for exceptions to this is that persons successfully completing a given education often try for a year or more at a more advanced school activity, but drop out or fail. In many cases, it is then reasonable to classify them as having the education which they successfully completed last. Furthermore, many roads through a school system may lead to a given final education. For these reasons, each column of R may contain several positive elements.

Furthermore, since we assume that it is possible to go from a given school activity to any of several different absorption categories (depending for instance upon whether the individual fails or succeeds in an exam); the matrix R may have several elements on each row.

If a person is in a category representing one of the E completed educations, we assume that he is "absorbed" in the sense that he will not leave that state. This assumption could be modified by introducing possibilites for returning to certain types of schools after some years of work (adult training, retraining, etc.). If we want to treat such cases satisfactorily, we must use a somewhat more general model than the one outlined here. A modification of our model permitting a crude treatment of such cases is, however, given in our empirical application (cf. the concept of "intermediate years").

The row sums of Q and R combined add up to unity, because a person may either go to one of the S school activities or leave school, thus ending up in one of the E final education categories :

$$\sum_{s=1}^{S} q_{hs} + \sum_{e=1}^{E} r_{he} = 1 \quad (h=1,\dots,S). \tag{7}$$

We may write the matrix of transition ratios in the form of (2). The absorbing states (representing various types of finished educations) are put first, and all those in such a state stay in that state (cf. the unity matrix in the upper left-hand corner).

It is important to notice that the transition ratios for pupils in a given school activity are assumed to be equal, regardless of whether these pupils have taken the same or different paths through the school system in previous years.

II. Some implications of the model

A number of conclusions can be drawn on the basis of the above model. We shall give most of the results without proof, but as an example, we give the derivation in one case.

We must first find a formula for the fraction of the pupils now in a certain school activity (h) who will be in another specified activity (k) after τ years (τ = 1,2,...). Here h and k may represent any pair of activities.

We define the following transition ratio of order τ :

$q_{hk}^{(\tau)}$ = fraction of persons now in school activity h who will be found in school activity k in τ years (τ = 0,1,2,...).

3

By definition, $q_{hk}^{(0)} = 1$ if h=k and zero otherwise, and $q_{hk}^{(1)} = q_{hk}$. Furthermore

$$q_{hk}^{(2)} = \sum_{s=1}^{S} q_{hs}q_{sk} \quad ; \quad q_{hk}^{(3)} = \sum_{s=1}^{S} q_{hs}^{(2)}q_{sk}.$$

The following recursive formula is clearly valid:

(8) $$q_{hk}^{(\tau)} = \sum_{s=1}^{S} q_{hs}^{(\tau-1)}q_{sk} \qquad (h,k = 1,\ldots,S)$$

The typical element of this sum, $q_{hs}^{(\tau-1)}q_{sk}$, gives the fraction transferring from h to s in $(\tau - 1)$ steps times the fraction transferring from s to k in one step.

The expression $q_{hk}^{(2)}$ is element (h,k) of the matrix Q^2. Similarly, using the recursive formula, we find that $q_{hk}^{(\tau)}$ is element (h,k) of the matrix Q^τ. Therefore :

(9) Element (h,k) of the transition matrix Q to the τ'th power gives the fraction of pupils now in school activity h who τ years later will be in school activity k ($\tau = 0,1,2,\ldots$).

The fraction of pupils in school activity h who will τ years later be found in any of the S school activities can also be found. This fraction is called a school staying ratio, and denoted by $q_h^{(\tau)}$,

(10) $$q_h^{(\tau)} = \sum_{k=1}^{S} q_{hk}^{(\tau)} \qquad (h=1,\ldots,S),$$

that is the sum of row h of the matrix Q^τ .

Without proof, I add the following results (each result is given a formula number for ease of reference):

(11) The average time spent in the future in school activity k by those now beginning activity h, to be denoted by w_{hk}, is element (h,k) of the matrix $W = (I-Q)^{-1}$, which is the inverse of the difference between the identity matrix and the transition matrix Q.

(12) The average number of school-years remaining for a person now at the beginning of school activity h is equal to the sum of row h of the matrix $W = (I-Q)^{-1}$.

(13) The fraction of pupils now at the beginning of school activity h who τ years later will graduate with final education e, to be denoted by $r_{he}^{(\tau)}$, is element (h,e) of the matrix $Q^{\tau-1}R$ (the transition matrix Q to the (τ-1)th power times the matrix R).

(14) The fraction of pupils now at the beginning of activity h having completed education e within x years is obtained by summing the above coefficients $r_{he}^{(\tau)}$:

$$\sum_{\tau=1}^{x} r_{he}^{(\tau)}.$$

This expression will be called a completion ratio.

(15) The fraction of pupils now in school activity h who will sooner or later leave the school system with education e, is equal to

$$\sum_{\tau=1}^{\infty} r_{he}^{(\tau)}.$$

It can be shown that this expression is element (h,e) of the matrix $(I-Q)^{-1}R$.

The proofs of all these results follow from the theory of absorbing Markov Chains (see in particular the book cited above, [5] [1]

The Markov Chain model used above can also be treated as a system of simultaneous difference equations. We shall here do this briefly. (It is not necessary to read the rest of this section in order to understand the empirical parts of the paper). The following additional symbols are required :

$P_s(t)$ = number of pupils in school activity s in school-year t

$y_s(t)$ = number of new entrants from outside of the system to school activity s in school-year t

$g_e(t)$ = number graduating with final education e at the end of school-year t.

It follows from our assumptions that

$$P_s(t) = \sum_{h=1}^{S} q_{hs} P_h(t-1) + y_s(t) \qquad (s=1,\ldots,S). \tag{16}$$

Final graduation at the end of year t (for education e) is given by

$$g_e(t) = \sum_{s=1}^{S} r_{se} P_s(t). \tag{17}$$

By recursive use of formula (16) we arrive at an expression for $P_s(t)$ in terms of previous "entrance" :

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[1] The proofs will be given in the final report about this investigation.

(18) $$P_s(t) = y_s(t) + \sum_{h=1}^{S} y_h(t-1)\, q_{hs} + \sum_{h=1}^{S} y_h(t-2)\, q_{hs}^{(2)} + \dots$$

$$(s=1,\dots,S).$$

It is therefore possible to study the pattern of enrolment and graduation for alternative developments of the number of new entrants. We shall briefly discuss two special cases: the stationary and the exponential ones.

Suppose first that $y(t)$ is equal to y^* for all t. The stationary solution for P_s (to be denoted P^*_s) is then found by solving the following equation system, where P^* is the column vector $(P^*_1 \dots P^*_S)'$, and y^* is column vector $(y^*_1 \dots y^*_S)'$:

(19) $$P^* = Q'P^* + y^*.$$

The solution is:

(20) $$P^* = (I-Q')^{-1}\, y^* = W'y^*,$$

where I is the identity matrix of order S. Here, the enrolment in each school activity, which is constant over time, is a function of annual initial enrolment in each school activity. Element (k,h) of W', which is element (h,k) of its transpose W, gives the number of pupils in school activity k per person entering activity h per year (cf. conclusion (11) above).

Furthermore, in the stationary case we get the following solution for annual graduation, using (17) and (20), and putting g^* for the vector $(g^*_1 \dots g^*_E)'$:

(21) $$g^* = R'P^* = R'(I-Q')^{-1}\, y^*$$

Element (e,h) of $R'(I-Q')^{-1}$ gives the fraction of pupils starting in school activity h who will end up with education e (cf. conclusion (15) above). The formula therefore gives annual final graduation from each school activity given the "initial" annual enrolments. In many countries only the first grade of the primary school has "initial enrolment", so that vector y may have only one positive element, the rest being zeros.

The second special case we will discuss is that of exponential development of primary enrolment. For simplicity we shall assume that all entrants come to activity N° 1, so that the only "entrance function" is

$$y_1(t) = Aa^t , \tag{22}$$

where (a-1) is the growth-rate, and A is a constant. From (18) and (22) we have

$$P_s(t) = Aa^t \left[q_{1s}^{(0)} + a^{-1} q_{1s}^{(1)} + a^{-2} q_{1s}^{(2)} + a^{-3} q_{1s}^{(3)} + \dots \right], \tag{23}$$

where $q_{1s}^{(0)} = 1$ if $s=1$ and zero otherwise. If $a=1$, it can be shown that this reduces to the stationary case, cf. (20).

The ratio of the number of pupils in each school activity to the annual entrance to activity N° 1 is a function of a and of the transition ratios:

$$\frac{P_s(t)}{Aa^t} = q_{1s}^{(0)} + a^{-1} q_{1s}^{(1)} + a^{-2} q_{1s}^{(2)} + a^{-3} q_{1s}^{(3)} + \dots \tag{24}$$

For given transition ratios, this fraction is smaller the higher the growth-rate of first-time enrolment (a-1). For elementary school activities the transition ratios of high order are small, so that the "discounting" implied by formula (24) does not count very much. For universities etc., however, the transition ratios, $q_{1s}^{(\tau)}$, of relatively high order are dominant, and here the "discounting" is much stronger. Therefore, the rate of growth of primary enrolment (y_1) will, given the transition ratios, affect the composition of the school system. If $a > 1$, the "lower" schools will dominate over universities, as compared to the stationary case ($a=1$).

III. Adaptation of the model for empirical use

Because the assumptions of the Markov chain model are fairly rigid, it gives, at best, a crude approximation to the educational pattern. We shall here give a brief survey of how we have adapted the model. The main "adaptation" consists in the classification of the various parts of the educational system into a number of activities of one year's duration. In many cases this is artificial, and many conventions have to be adopted. To a certain extent the difficulties resemble those encountered in the classification of many types of economic (and partly non-economic) activity in the national accounts. We shall discuss a few points.

Timing of entrance and graduation: Fortunately for our purposes, the school-year is standardized in a large part of the school system, starting in August or September and ending in May or June. For the purposes of our analysis we assume that examinations and entrance to schools take place in the middle of the calendar year. All pupils enrolled in the autumn are assumed to stay at school throughout the school-year, and drop-outs are treated as if they had left at the end of the school-year. The gradual process of pupils dropping out throughout the year is therefore not specified. The model is best suited to types of schooling organized in a fairly rigid way, with stages of one year's duration. It is not as well adapted to university studies where students may have considerable freedom in choosing the period of time between each major examination.

Part-time study: Some people attend school full-time, but for only part of the year. Others attend school only part-time, either for part of the year or for the whole year. These cases have simply been divided into two categories: some are treated as if they were in full-time education all year, and the rest are totally omitted from the model. A more satisfactory approach in a more comprehensive model would be to introduce special activities for each sub-category of part-time schooling.

"Intermediate years": Many students interrupt their education for some time and continue it later. The reasons can be involuntary, such as illness and compulsory military service. Furthermore, in some schools "practice" is an entrance requirement. Some may return to school for other reasons after a few years of work. As a crude approximation, in order to deal with such cases, we have chosen to introduce a number of "intermediate years" after each major exam, as separate activities. For example, a certain fraction of Secondary school graduates continue their schooling at once; others leave for good; and some leave the school system temporarily. The latter are here treated as entrants to the category "First intermediate year after completing Secondary school". In the next year, some of these go to some school or college, and the rest transfer to the category "Second intermediate year after completing Secondary school". And so on. The "intermediate years" are therefore treated somewhat artificially as school activities, and included among the S categories of the transition matrix Q. Because of this, care is needed in interpreting some of our results.

For a list of the "intermediate years" actually introduced into the model, see Table I.

Repetition: A weakness of our approach is connected with its "first-order nature" (it is formulated as a Markov chain of the first order). This implies, among other things, that the same transition ratios are used for newcomers to a school activity, and for first-year and second-year repeaters. Alternatively, we might have added special repetition activities in connection with each regular school activity. In a second-order Markov chain, transition probabilities are assumed to vary not only with the present state, but also with the previous state. Similarly, we might have introduced transition ratios specified by previous and present school activity. But such refinements are beyond the scope of the present crude approach.

Mortality: A convenient way of handling mortality in the model is to treat it as an absorbing state together with the completed educations. The last column of matrix R in (6) thus gives the fractions of pupils in the respective school activities dying during the year. In accordance with the rest of the model, we use a non-stochastic approach for mortality as well.

Types of education excluded: The present model treats only formal education, taking place in organized schools. Therefore, we have excluded the very important on-the-job training, as well as parents' teaching of their children, self-study, and study by means of correspondence courses.

We have also omitted courses of less than half a year's duration, as well as part-time training. Quite a few Norwegians study technical subjects abroad (below university level), but this type of training is excluded here. Furthermore, the special schools for the disabled, blind, deaf, etc. are excluded. Finally, in order to limit the size of the model, we have not included the first six years of compulsory school for children (Barneskole). The lowest educational activity included is consequently the 1st year of Secondary school (1st year of Ungdomsskole or 7th year of Folkeskole).

Table I gives a survey of our breakdown of the school system into 54 school activities and 6 "intermediate years".

Table I

School categories included in the model, with English and Norwegian names

Activity N°	Name used in this study	Corresponding Norwegian name
1-4	Secondary schools	Ungdomsskoler, realskoler, framhaldsskoler
5-7	Gymnas	Gymnas
8-9	Folk high schools	Folkehøgskoler
10-13	Teachers' training colleges	Alminnelige laererskoler
14-15	Special teachers' training colleges	Fag- og spesiallaererskoler
16-17	Vocational schools for agriculture	Fagskoler for landbruket
18-21	Workshop schools	Verkstedskoler
22-25	Other vocational schools for handicraft and industry	Andre håndverks- og industriskoler
26-27	Maritime schools	Maritime skoler
28-30	Vocational schools for the service occupations	"Tjenesteytingsskoler"
31-37	Universities A ("closed")	Universiteter og høgskoler, tradisjonelt "lukkede" studier
38-47	Universities B ("open")	Universiteter og høgskoler, tradisjonelt "åpne" studier

Activity N°	Name used in this study	Corresponding Norwegian name
48-54	Foreign universities	Universiteter og høgskoler i utlandet
55-56	Intermediate years after completing elementary school	"Mellomår" etter folkeskolens 7. klasse eller etter framhaldsskolen
57-58	Intermediate years after completing Secondary school	"Mellomår" etter fullført realskole eller ungdomsskole
59-60	Intermediate years after the Gymnas	"Mellomår" etter fullført examen artium

1) It turned out to be difficult to give an English translation of some of the Norwegian school names. This is in particular the case for the Gymnas, which is perhaps equivalent to the last one or two years of an American High school and the first one or two years of Junior College. I have chosen to use the Norwegian term "Gymnas" in this paper.

The fairly large number of activities permitted classification according to level as well as a crude classification according to subject of study.

We have split the universities into two categories, called A and B. Category A contains universities which have been practising rather strict admission control for some time. Category B contains universities or parts of universities where any one successfully passing the examen artium is permitted to enter. The main reason for this separation is that the drop-out pattern and length of study are very different for the two categories. However, from the point of view of practical application, a breakdown according to field of study would have been of greater interest.

Final educations are classified into 17 levels (see Table II), and an additional category (N° 18) represents mortality.

Table II

Final educational categories of the model, with English and Norwegian names

Education N°	Final education (in some cases we use the name of the highest school completed)	Corresponding Norwegian name
1	Primary school	Folkeskole
2	Continuation school	Framhaldsskole
3	Secondary school	Ungdomsskole, realskole
4	Gymnas	Gymnas
5	Folk high school	Folkehøgskole
6	Teacher training	Alminnelig laererskole
7	Special teacher training	Spesiallaererskole
8	Vocational school for agriculture	Landbruksskole
9	Workshop school	Verkstedskole
10	Other vocational schools for handicraft and industry	Andre håndverks- og industriskoler
11	Maritime school	Maritim skole
12	Vocational school for the service occupations	"Tjenesteytingsskole"
13	University A ("closed")	Universiteter og høgskoler, "lukkede"
14	University B ("open"), lower degree	Universiteter og høgskoler, "åpne", deleksamen

Education N°	Final education (in some cases we use the name of the highest school completed)	Corresponding Norwegian name
15	University B ("open"), higher degree	Universiteter og høgskoler, åpne, avsluttende eksamen
16	Foreign university, lower degree	Universiteter og høgskoler i utlandet, deleksamen
17	Foreign university, higher degree	Universiteter og høgskoler i utlandet, avsluttende eksamen

The classification follows to a large extent from our classification of school activities. Explanation is perhaps needed on a few points.

Consider first educational categories 1 - 2 - 3. Among the activities included in what we have called, in Table I, Secondary school (activity N° 1) is the last (7th) year of Primary school. In 1961/62 this was still the final education for some pupils, here represented by our final education category N° 1 (Primary school). Other pupils continued their schooling for one or two years after Primary school in so-called Continuation schools ("framhaldsskoler"), and those finishing their education on leaving such schools have been classified into educational category N° 2 (Continuation school). Those terminating their education by either completing the new "ungdomsskole" or the "realskole" have been classified into category N° 3 (Secondary school).

For university graduates, we have distinguished between those terminating their studies with a lower degree (usually after about three to four years of study) and those obtaining a final degree (usually after about five to seven years of study).

It should perhaps be stressed again that a person reaching one of the 17 final education categories is assumed to stay in that category. Thus in this model we do not introduce any assumptions about labour force participation, mortality, etc.

IV. Norwegian data on the educational pattern

Our task is now to arrive at estimates of Norwegian transition ratios, and to use these ratios as a basis for giving numerical results corresponding to the mathematical results given in (9) - (15) of Section II.

The data was collected by Kaare Andersen [1], but all the main principles involved (sector classification, treatment of special types of schools and educations, etc.) were decided upon after discussions with me. The data consists of :

(a) Number of pupils in each of the school activities in 1961/62 and 1962/63.
As a rule, the numbers of pupils on October 1, 1961 and on October 1, 1962 were used to represent the numbers of pupils in each of the two school-years, respectively.

(b) Flows of pupils between each pair of school activities between 1961/62 and 1962/63, and final graduation from each school activity to each final education category at the end of school-year 1961/62.

The structure of the table containing our data is shown in Table III below. The symbols in Table III are defined as follows ($P_s(t)$ and $g_e(t)$ are defined in Section II) :

$P_{hk}(+)$ = flow of pupils from school activity h to school activity k between the years t and t+1.

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[1] Consultant at the Norwegian Research Council for Science and the Humanities.

$g_{he}(t)$ = final graduation of pupils from school activity h with final education e at the end of school-year t.

Table III

Structure of the main data table of flows between the S + E activities

From school activity	To school activities		To final educations[1]		Number of pupils in year t
	1	2 . . . . S	1	2 . . . . E	
1.......	$P_{11}(t)$	$P_{12}(t)...P_{1S}(t)$	$g_{11}(t)$	$g_{12}(t)...g_{1E}(t)$	$P_1(t)$
2.......	$P_{21}(t)$	$P_{22}(t)...P_{2S}(t)$	$g_{21}(t)$	$g_{22}(t)...g_{2E}(t)$	$P_2(t)$
.	.	. .	.	. .	.
.	.	. .	.	. .	.
S	$P_{S1}(t)$	$P_{S2}(t)...P_{SS}(t)$	$g_{S1}(t)$	$g_{S2}(t)...g_{SE}(t)$	$P_S(t)$
From the "outside"	$y_1(t+1)$	$y_2(t+1)...y_S(t+1)$			
Sum.....	$P_1(t+1)$	$P_2(t+1)...P_S(t+1)$	$g_1(t)$	$g_2(t)$ ...$g_E(t)$	

[1] Including mortality as activity N° E.

In the present case, S = 60, E = 18 (including mortality), year t represents school-year 1961/62 and (t+1) the next school-year.

It was not possible to extract all the elements of this table from Norwegian educational statistics. Kaare Andersen, an expert in using our educational statistics, had instead to rely on the following type of approach :

(a) First, the number of pupils in 1961/62 and 1962/63 in each of our school activities was estimated. As a rule, fairly reliable estimates could be obtained.

(b) Next, using various sorts of information available from our educational statistics, a number of the elements $P_{hk}(t)$ and $g_{he}(t)$ (t=1961/62) were estimated directly.

(c) In order to estimate repetition, $P_{hh}(t)$, various investigations into repetition in Norwegian schools were used.

(d) The remaining elements of the table were estimated partly by using miscellaneous statistical information, partly by "informed guessing", always using the row and column totals as a check.

A detailed survey of the sources used is given in a mimeographed paper by Kaare Andersen.[1] As mentioned above, parts of the data are far from reliable. However, instead of waiting for years for better flow statistics for our educational system, the present author has chosen to build upon whatever empirical information could be pieced together to form a comprehensive picture. The outcome may at least serve to illustrate some methodological principles. And it may even serve as a stimulus for the improvement of the educational statistics.

Because of the paucity of data, the "cross-roads" through the educational system are probably underestimated. Combinations of different types of vocational training are perhaps more common than indicated by our tables, but better educational statistics are necessary in order to arrive at more realistic estimates on this point.

In this paper we omit the detailed tables containing the data (the complete data and numerical results will be given in the final report on this investigation).

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[1] See "Vedlegg II" in [11].

V. Derived characteristics of the Norwegian educational pattern

The transition ratios q_{hk} and r_{he} of (3) and (4) were estimated, using the following formulas:

$$q_{hk} = \frac{p_{hk}(t)}{p_h(t)} \qquad (h,k = 1, \ldots, 60),$$

$$r_{he} = \frac{g_{he}(t)}{p_h(t)} \qquad \begin{array}{l}(h = 1, \ldots, 60;\\ \ e = 1, \ldots, 18).\end{array}$$

The coefficient matrix $\left[\, Q \mid R \,\right]$ is of order 60 x 78, a rather unmanageable size, and shall not be reproduced in this preliminary report.

We shall briefly discuss decomposition of the matrix Q. By permuting the sequence of the activities, and by putting intermediate years 55 - 58 between activities 9 and 10, we obtain a matrix of the following structure:

$$Q = \left[\begin{array}{c|c} Q_{11} & Q_{12} \\ \hline 0 & Q_{22} \end{array}\right]$$

Here Q_{11} is a square matrix of order 13 representing internal transitions among the set of activities 1 - 9 and 55 - 58. Similarly, Q_{22} is a square matrix of order 47, and represents internal transitions in the set of activities 10-54 and 59-60. We may say that activities Nos 1 - 9 and 55 - 58 represent general education, and the rest vocational training, both taken in a very broad sense.[1] In our data, the transfer from the former to the latter is represented by Q_{12}, while there is no recorded transfer in the opposite direction.

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[1] The "Intermediate years" are here divided between these two broad categories.

We might also have decomposed matrix Q_{22}, and it is possible to single out several sub-chains, from which pupils do not transfer to any school activities outside the sub-chain.

Inspection of the matrix Q reveals that complete triangularization is impossible, but the matrix is not far from being triangular, in the sense that the activities can be arranged in such an order that only a few relatively small elements are found below the main diagonal. In our computations, we have treated Q as an entity, without using the simplifications which could have been obtained by taking the above mentioned qualitative characteristics into account.

Many of our final educations are heterogeneous in the sense that the duration of stay in the last school before graduation is rather different. As an example, final education N° 12 consists of pupils taking either only the first, the first and the second, or all three stages of Vocational school for the service occupations. In a more detailed model, it would probably be important to classify these three levels in different categories. Furthermore, this broad category of schools is in itself very heterogeneous with respect to topics, level of teaching, etc..

Anyone knowing the present-day educational situation in Norway will realise that some of our transition ratios are already (in 1966) somewhat out of date. One reason for this is the structural change in the school system, switching gradually from 7 to 9 years of compulsory education. In 1961/62, the new system was only adopted in very few communities. It would be possible to guess how these institutional changes will effect the transition ratios, and then compute the future implications of this new set of transition ratios.

On the basis of our coefficient matrices R and Q, the following matrices were computed (using an electronic computer):[1]

$$Q^{\tau}, \text{ for } \tau = 2,3 \ldots, 10, 15, 20.$$

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[1] The computations were financed by "Metodeutvalget for arbeidskraft-prognoser", Norwegian Directorate of Labour.

$W = (I - Q)^{-1}$, and row sums of W.

$(I - Q)^{-1} R$.

Due to the size of our matrices, it is obvious that we cannot present all our results. We shall therefore confine ourselves to illustrative examples of the main types of results. The results will be arranged so as to correspond with theoretical results (9) - (15) of Section II.

Higher order transition ratios (cf. [9])

I have computed higher order transition ratios for each starting activity (cf. (9)). These detailed results are omitted here. I should like to mention, however, that the ratios of order 20 are practically all zero, as could be expected. For example, twenty years after staying in the last year of Secondary school practically everyone has left the school system.

A much less detailed, but perhaps more readable, picture of the transition process is given in Table IV. This table illustrates the flow of an initial group of pupils through the main parts of the school system. The peak load on the Gymnas comes after about 4-5 years on the Folk high schools after 4 years, and on the Teacher training colleges after 7 years (there is a wide dispersion over time). The peak load on the vocational schools occurs after 4 years, with 18.11 per cent of the cohort attending these schools. The university enrolment is very low in the first five years, but reaches a level of 7.40 per cent of the group after 8 years.

School staying ratios (cf. [10])

The school staying ratios for activity N° 1 are given in the bottom line of Table IV. Notice that after 5 years one third of the pupils are still in school, after 8 years only one out of eight of the initial pupils remain at school, and after 15 years, when their age is about 28 - 29 years, only 1.24 per cent are left in the educational system.

For the pupils in the first grade of the Gymnas (activity n° 5) the school staying ratios are higher. After 5 years almost 55 per cent are still in school, and after 8 years one fourth still remain. Even after 10 years, as many as 13 per cent have not yet left the educational system.

Time spent in each school activity (cf. [11])

We shall only give a few scattered results here. Pupils in activity N°1

Table IV

Percentage of pupils in the first year of Secondary school (activity N° 1) who will be in each of the main school categories n years hence

Transition ratios $q_{1k}^{(n)}$ aggregated over school activities (k)

Acticity N°	School category	n=1	n=2	n=3	n=4	n=5	n=6	n=7	n=8	n=9	n=10	n=15	n=20
1-4	Secondary schools	97.37	59.29	29.49	6.05	1.69	0.49	0.10	0.02	0.01	-	-	-
5-7	Gymnas	-	-	15.85	17.09	17.63	5.61	1.80	0.46	0.12	0.02		
8-9	Folk high schools	-	2.13	1.72	3.38	1.32	0.26	0.07	0.02	-	-	-	-
10-15	Teacher training colleges	-	-	0.08	1.10	1.26	2.95	3.76	2.05	1.01	0.35	-	-
16-30	Vocational schools	-	2.33	10.99	18.11	11.06	10.53	5.19	2.82	1.07	0.38	-	-
31-54	Universities	-	-	0.02	0.09	0.24	3.95	6.18	7.40	7.37	6.73	1.24	0.04
Percentage of pupils in school after n years..............		97.37	63.75	58.15	45.82	33.20	23.79	17.10	12.77	9.58	7.48	1.24	0.04

spend on the average about 0.19 years in each of the three grades of the Gymnas, a total of 0.42 years in domestic universities, and 0.08 years in foreign universities. Those at the beginning of the last grade of the Gymnas spend, on the average, 1.08 years in their present school activity, about 0.19 years in each of the last two grades of Teacher training colleges, and 0.46 years in Vocational schools for the service occupations. They spend on the average 2.09 years in domestic universities, and 0.38 years in foreign ones.

Average number of school-years left for pupils at the beginning of a given school activity (cf. [12])

On the average, slightly less than five school-years are left for pupils at the beginning of activity N° 1, normally at the age of 13 or 14 years. However, for a pupil in the first grade of the Gymnas (normally aged 16), on the average almost 6½ years are left. For students enrolling in the different categories of universities, the average remaining duration of education is from 4.75 to 6.11 years.

Time-specific graduation ratios (cf. [13])

The results show that the time lag between first grade of Secondary school and final graduation with a given education is by no means unique. This is partly due to repetition, partly to the existence of "intermediate years", and partly to the fact that some final educations can be reached via different "routes". Teacher training colleges can for instance either be entered from Secondary school or from the Gymnas - in the latter case pupils enrol in what we have called the third grade of Teachers' college. A further reason for the wide dispersion of times of graduation is that some of our final education categories are very heterogeneous; some of them contain, for instance, graduates with short as well as long vocational training.

Completion ratios (cf. [14])

We have computed the fraction of pupils now at the beginning of a given school activity who will have completed a certain education within x years (x=1,2...). Here we shall only give the results for pupils in school activity N° 1, and for these we only consider education N° 6, Teacher training.

Table V

Completion ratios

Percentage of pupils at the beginning of 1st grade of Secondary school who will have completed Teacher training college within x years.

x	Completion ratio	Completion within x years as a percentage of final completion
7................	0.02	0.56
8................	1.87	52.97
9................	2.79	79.04
10................	3.30	93.48
11................	3.47	98.30
...	....	.....
Limiting value	3.53	100.00

The completion ratios could have been used as a sort of test of the model, by comparing them with other information, for instance age-limits for admission to the various schools, age-distribution of graduates from the different schools, etc . Such checks have not yet been undertaken systematically.

Final educational distribution of the pupils in a given school activity (cf. [15])

Of the pupils in the 1st grade of Secondary school, about 48 per cent obtain only general education, about 40 per cent vocational training, 4.5 per cent teacher training, and almost 8 per cent receive a university degree. Among the pupils of the final grade of the Gymnas, about 39 per cent complete university education successfully, while approximately 17 per cent obtain teacher training.

It is of interest to compare the educational distribution arrived at in this way with the results of the population census of 1960 (see Table VI). In comparing the results, one must be aware that the census gives the educational distribution of the stock of people, whereas our results in the first column of the table refer to annual flow of graduates under conditions of stationary intake of pupils to activity N°1.

In the census, people were asked to report all completed exams and tests normally requiring at least one school-year of preparation. It should be observed that in the table we have used the term "vocational training" to denote all non-general training, except teacher training and university training. It is probably questionable to equate the educational categories of the census with those of our model, partly because of changes in the content of the different educational categories with time.

The most remarkable differences between our results and the census data concern teacher training and university training (see Table VI). The differences imply that present educational propensities are leading towards an educational distribution of the population very different from that of 1960.

When we consider different age-groups, we see that the oldest group had the lowest fraction of university-trained people, viz. 1.41 per cent. The highest fraction, 2.47 per cent, was found in age-group 30-39 years. The lower rate for the 25-29 year age-group is probably due to the fact that some of the persons in this group had not yet completed their university education. As can be seen, according to our transition ratios the fraction of university-trained is more then three times the fraction in the age-interval 30-39 years (which has the highest fraction of any group).

Scattered statistical evidence leads me to believe that if we had used 1964-65 data instead of 1961-62 data, the observed increase would have been still stronger.

Table VI

Comparison of final educational distribution of a group of pupils according to our transition ratios with the observed educational distribution of the population in the 1960 population census

Educational categories	Final educational distribution of pupils in 1st year of secondary school	Educational distribution for persons in the following age-intervals, according to the population census of 1960 2)					
		25 - 69	25 - 29	30 - 39	40 - 49	50 - 59	60 - 69
General education only...	47.67	77.28	69.35	71.35	77.43	81.69	84.78
Teacher training.........	4.48	1.14	1.73	1.27	0.63	1.09	1.42
Vocational training......	39.79	19.61	26.92	24.91	19.98	15.38	12.39
University education.....	7.74	1.97	2.00	2.47	1.96	1.84	1.41
Total....................	99.68 1)	100.00	100.00	100.00	100.00	100.00	100.00

(1) Mortality during schooling is excluded.

(2) Population Census 1960, Vol. IV, N° XII 133, Central Bureau of Statistics of Norway, Oslo 1964.

Forecasts of school attendance and graduation (cf. [16] and [17])

Using the observed matrices Q and R, data describing the initial situation with respect to pupils, and estimates of the number of new entrants to the school system every year, the model has been used for forecasting (see formulae (16) and (17)). The actual outcome of these forecasts is of only limited interest, since we know that the transition ratios will change over time, partly because of capacity limits. Nevertheless, conditional forecasts of this type may serve as a first orientation for the practical forecaster. I have computed annual forecasts for enrolment in each of the 60 school activities, and final graduation from each of the 17 education categories from 1963-1975, but I will not present the numerical results here.[1]

VI. Final remarks

The analysis reported in this paper could be improved and extended in many ways.

In addition to our final graduation coefficients (r_{se}) we have introduced and estimated another type of graduation coefficients, expressing the fraction of pupils in a given school activity who at the end of the year will successfully graduate with a given education. This fraction also includes those who transfer to other school activities in the next year. We could also compute the teacher requirements corresponding to given initial annual enrolments, and check whether enough teachers are educated to make possible the transition pattern implied by the model. Such results will be given in the final report.

The analysis so far hinges on the stability over time of the transition ratios. Recently, however, we have been in the midst of an "educational explosion"in Norway, characterized by rapidly changing transition ratios. If we know such changes in advance, it is relatively easy to study the transition process, although the simple elegance of this type of Markov chain theory is lost. So far, little seems to be known about the factors quantitatively determining the transition ratios.

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[1] The results are given in a mimeographed paper [11] (in Norwegian).

In sociological investigations the influence of parental social class and education upon the propensity to study is emphasized. Separate transition matrices could be used for pupils coming from different social and geographical environments, and the influence of changes in the parental occupational and educational structure could thus be accounted for.

In this model we have not taken the intelligence level of students explicitly into account. It would perhaps be better to study the schooling patterns of different intelligence categories separately.

Another interesting question is to what extent we can influence transition ratios by means of various types of educational policy. Economic measures, for example increasing scholarships in certain schools, may influence many transition ratios, and produce final effects which can only be analysed by means of a comprehensive educational model.

The most important objection to the transition type of analysis is probably that the transition ratios are determined by pupils' behaviour as well as by the rules and decisions of educational institutions (enrolment policy for instance). To take an analogy from economic theory the transition ratios are neither "demand" nor "supply" parameters, but rather a mixture of both. The transition ratios may for instance turn out to be very sensitive to changes in the behaviour of the school authorities.

I have tried to modify the present model by explicitly introducing capacity constraints in some of the school activities. Assumptions about the behaviour of "overflow" pupils who do not obtain admission to the school where they apply are then also introduced. It is possible to make a simulation model which takes capacity constraints explicitly into account.

For educational planning, it would have been of considerable interest to reverse the problem of this paper by asking how the school system should be adapted in order to satisfy given manpower needs.[1]

As a part of this, one might ask what the transition matrix should be for a desired composition of the output from the school system. In my paper 10 I have worked out a solution to this latter problem under special assumptions.

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[1] See J. Tinbergen and H.C. Bos [8], and my own paper [10].

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THE DEVELOPMENT OF COMPUTABLE MODELS OF THE BRITISH EDUCATIONAL SYSTEM AND THEIR POSSIBLE USES

by Peter Armitage and Cyril Smith

The project to develop computable models of the British educational system is being jointly carried out by the Department of Education and Science and the Unit for Economic and Statistical Studies on Higher Education.* The beginnings and nature of the project have been described in papers by Professor C.A. Moser, the Director of the Unit, and Mr. Philip Redfern, Chief Statistician of the Department. (1,2)

1. Introduction

In recent years there has been a dramatic upsurge in interest in the development of the educational system. This has found expression in such reports as those of the Robbins Committee whose minute of appointment read as follows:

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* The Unit for Economic & Statistical Studies on Higher Education is a part of the London School of Economics and Political Science. It was established in 1964 with a grant from the Nuffield Foundation. Individual projects are financed by specific grants and the Department of Education and Science supports the Unit with a grant for the Model Project.

> "To review the pattern of full-time higher education in Great Britain and in the light of national needs and resources to advise Her Majesty's Government on what principles its long-term development should be based. In particular, to advise, in the light of these principles, whether there should be any changes in that pattern, whether any new types of institution are desirable and whether any modification should be made in the present arrangements for planning and co-ordinating the development of the various types of institution."

As the appendices to the Robbins Report show, it was necessary and relevant to sift a vast body of statistical data and also to make assumptions about the future development of the system in order to fulfil these terms of reference. In the same way, the National Advisory Council on the Training and Supply of Teachers has required estimates of the school and teacher populations in the future in order to study "a persistent and deep rooted problem" and to advise on possible remedies for alleviating the shortage of teachers. Because of these and similar studies the Statistics Branch of the Department of Education and Science has increasingly been called upon to provide projections of parts of the educational system based on a variety of assumptions. As these demands grew, it was natural that attention should be directed to developing a general computable model which would facilitate calculations of this kind and have the virtues of consistency and coherence.

Over the last twenty years there have also been comparably dramatic developments in computers and in the mathematical sciences with the development of many new techniques. These have been successfully applied in many different areas of industry and business and recently there has been a concentration of interest in model-building. At one end of the scale there have been econometric models which represent highly complex systems. The practical value of these models may be limited, since they can only represent the empirical relationships with fairly simple mathematical equations. At the other end of the scale there have been models of industrial processes where the systems are less complex and more is known about the functional relationships embodied in them. In these cases the flow of information can be so organised that the system is progressively better understood, and the model can become a highly efficient means of plant control. The latter applications suggest a much wider scope for a model of the educational system than the improvement of projections. However, the study of socio-economic systems presents rather different problems with new features which will be discussed towards the end of this paper.

For whatever purpose the model may eventually be used, we are concerned in the first instance with the setting up of a basic language of description. It is expected that we will wish to study alternative model structures, and when studying a particular model, allow its structure to change over time. We will also wish to vary the assumptions and parameters embodied in the model and to carry out repeated large-scale calculations with economy and rapidity. It is, therefore, necessary that our basic language should be both highly general and flexible in character.

2. The basic descriptive language of model structures

The general definition of a system is "a complex whole, a set of connected things or parts". Mathematical definitions emphasise the interrelationships and the fact that all the information needed to describe the system can be contained in a set of variables so that the vector of these variables describes the state of the system at a particular time.

The real educational system consists of individual students, teachers, schools and institutions, and equipment ranging from expensive machinery down to pieces of chalk. These can be regarded as the elements of the real system and their behaviour is familiar: a student changes his grade at school or gains the qualifications which enable him to progress to a higher institution; a teacher continues in the same position or changes schools or leaves the profession; schools are built or extended or closed; and so on. An individual student's progress is determined by many factors: for example, his natural ability, his education so far, the place where he lives and the facilities it provides, parental income and attitudes. Teachers too are individuals and every school is different, so that the real situation is found to be virtually inexhaustible in detail when it is examined closely. The consequence of this richness of detail is that limits must be placed upon the degree of detail included in a particular representation of the system. It may not be clear what is important and what is unimportant, but the detail included in the model is selected by judgment in the light of the purposes for which it is to be used, and for different purposes different degrees of detail will be appropriate. The wealth of detail also raises the question of whether it is better to adopt a micro or a macro approach and this, too, is a matter of opinion which will depend upon purposes.

It is important to distinguish between the real system and the model as an abstraction and idealisation of it. For the purposes of this paper we will adopt a macro approach, and confine ourselves to the educational system of England and Wales and to students and teachers. The level of aggregation adopted will be national, though models could be devised which would represent regional or local situations in the same manner.

It is now convenient to introduce some notation. Let us use $n(r,t)$ to denote the number of people in process r at time t. The processes may be thought of as the broadest categories corresponding to educational sectors, such as primary, secondary, university,etc., though they will usually be more detailed. Thus r may be the second year students of the sixth form at secondary grammar schools, or first year science students at universities, or teachers at primary schools. The notation can be extended so that $n(r,a,t)$ denotes the number of people aged a (a taking integral values 0,1,2,..... of years) in process r at time t. These numbers represent the stocks which we wish to distinguish, and it follows that the total number of people in the system at time t is

$$N_t = \sum_r n(r,t).$$

Some part of the stock $n(r,t)$ is due to new entrants to the system during the time period $(t-1)$ to t (i.e. births plus immigrants to the country to which the model relates), and we can denote this part by

$u(r,t-1)$. If we define $U_t = \sum_r u(r,t)$, then $N_t = N_{t-1} + U_{t-1} - D_{t-1}$,

where D_{t-1} take over the number of deaths and emigrants between time $(t-1)$ and time t.

In the same way, we may define flows of people so that $f(r,s,t)$ denotes the number of people in process r at time t who move to process s at time $(t+1)$. This leads to the two further identities:

$$n(r,t) = \sum_s f(r,s,t) \quad \text{and,}$$

$$n(s,t+1) = \sum_r f(r,s,t) + u(s,t).$$

It will be noted that we do not need a term in this equation for deaths and emigrants because deaths occur to people who are already in boxes in the system. New entrants are not and so must be represented by the additional term. When a student or teachers stays in the same process over successive time period, $r = s$. Since most educational activities work in an annual cycle, the time unit used is one year.

Clearly the flow of people moving from process r to process s will depend upon the number of people passing through the system, and it would often be misleading to study time series of actual flows expressed as absolute numbers. Consequently it is desirable to define the transition proportion $p(r,s,t) = f(r,s,t)/n(r,t)$, i.e., the proportion of people in process r at time t who move to process s at time $t+1$. It follows that

$$\sum_s p(r,s,t) = 1 \quad \text{and}$$

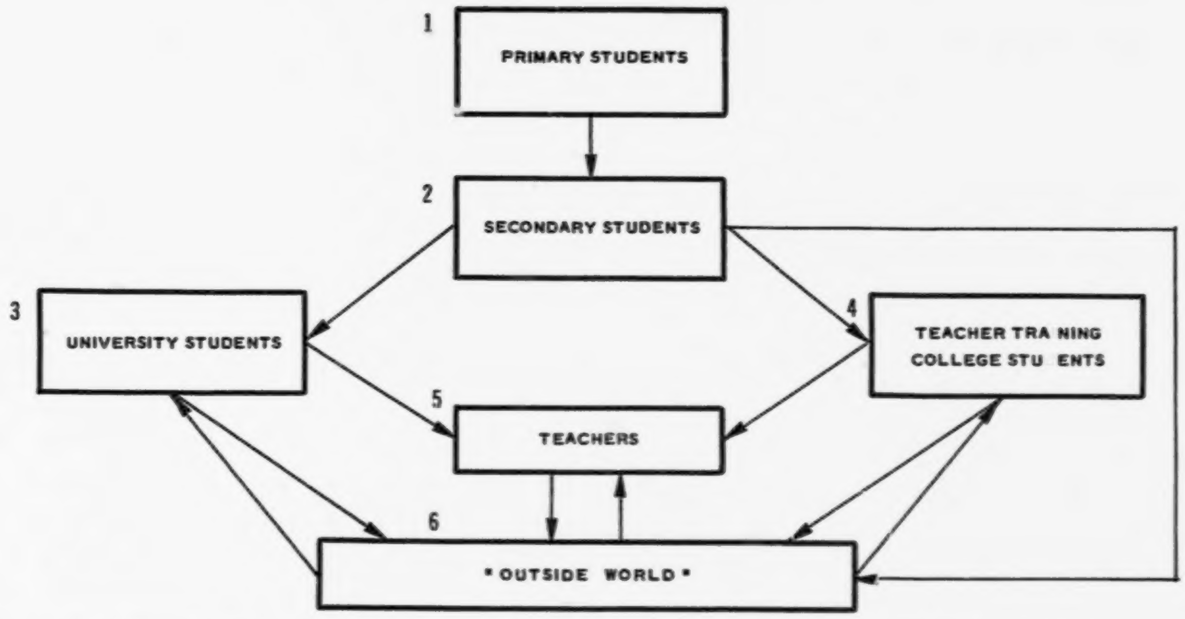
$$n(s,t+1) = \sum_r p(r,s,t)n(r,t) + u(s,t) \qquad (1)$$

This last recurrence formula (1) allows the future state of the system to be forecast provided the values of all $n(r,t)$ for base year t are given together with values of all u(s,t) and all p(r,s,t) for all required years.

This amounts to saying that we can forecast future states of the system if we know accurately its present state and are prepared to assume what movements are going to take place over the period of interest. Alternatively, if all values of n(r,t+x) are given, rather than values of n(r,t), we can work backwards to find n(r,t). In other words, if we set targets, and all is presumed known about movements and migration for the intervening period, we can work back to find what stocks we should have now in order to meet these targets, and so concentrate attention on the discrepancy vector of the differences between actual and desired stocks in year t. However, these calculations are possible only because we are either given or have presumed everything we need to know and because the system is still highly simplified. This is no longer possible when the system is made more complex.

We do not propose to introduce complexities at this stage, but rather to clarify the nature of our computer programmes. This is best done by a simple example. Suppose we are interested only in students at primary schools, secondary schools, universities and teacher training colleges, in teachers and in people outside the educational system, i.e. people who are currently neither students nor teachers. (The six processes identified are shown in Figure 1 as boxes). A possible set of movements between processes can be summarised by the flow diagram in Figure 1.

Figure 1
FLOW DIAGRAM OF A SIMPLE MODEL



Note : All "boxes" can be reached by new entrants from outside the system, i.e. births and immigrants

Movement is possible within all boxes so that the total movements can be summarised as in Table 1.

TABLE 1

Summary of movements in simple model

(Box numbers as in Figure 1)

Destination list		Origin list	
From	To	To	From
1	1,2	1	1
2	2,3,4,6	2	1,2
3	3,5,6	3	2,3,6
4	4,5,6	4	2,4,6
5	5,6	5	3,4,5,6
6	3,4,5,6	6	2,3,4,5,6

The destination and origin lists shown in Table 1 are, of course, equivalent. Initially only the origin list is used in the computer programme, but it will be convenient later to refer to the destination list. It is often convenient when constructing a model to set out the possible movements in the form of a matrix as in Table 2.

TABLE 2

Matrix of movements in simple model

Box numbers

From	To					
	1	2	3	4	5	6
1	Y	Y	N	N	N	N
2	N	Y	Y	Y	N	Y
3	N	N	Y	N	Y	Y
4	N	N	N	Y	Y	Y
5	N	N	N	N	Y	Y
6	N	N	N	Y	Y	Y

Y = Movement possible
N = Movement not possible

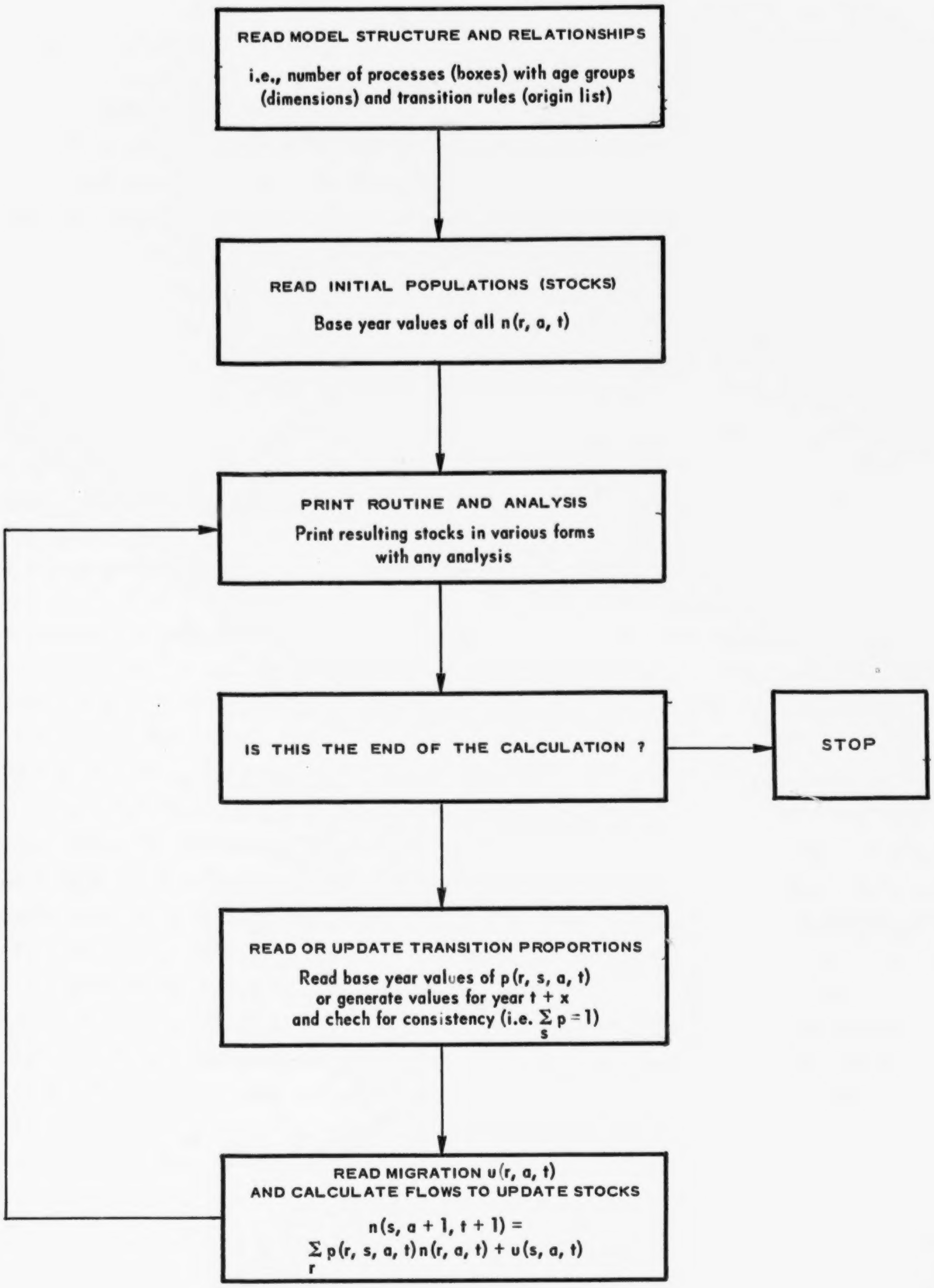
The generality and flexibility of the treatment is illustrated by this simple example. Any model is the description of a structure and its relationships; the structure consists of a number of identified processes which are connected by rules of movement which must be specified. The simple model described above consists of 6 boxes and 18 possible movements. Figure 1 could describe either the national or regional or local situation. It is not difficult to imagine a flow diagram similar to that in Figure 1 for a local education authority where each box represented a school in the area rather than a stage of education. Further detail may be introduced with ease by adding more boxes with all their connections to other boxes. It will be usual to break the boxes down into cells corresponding to single ages. When age is introduced in this way, the possible movement from one box to another becomes a vector with its number of elements determined by the age dimensions of the two connected boxes. For example, if secondary students (box 2) can be any age between 10 and 19, and university students (box 3) are aged 17 to 35, then f(2,3,t) is a flow vector with four elements since a secondary student in year t cannot move to university in year (t+1) until he is 16 (arriving in university at 17) and he can be no older than 19. The number of elements in every flow vector (in this example 4) is worked out in the programme. The scale and frequency of the calculations which we will want to carry out makes the use of a large computer inescapable, and it will be appreciated that this formulation of model structures is well suited to computer programming. There is great similarity between the language of the computer (the programming code) and our language of model structures.

A highly simplified flow diagram of the computer programme is shown in Figure 2.

Little further comment upon Figure 2 is needed except to point out that the updating of transition proportions may involve complicated procedures, and this is a key point at which complexity and realism can be introduced into the model. In the programme the updating is carried out in a sub-routine of which there can be several alternative versions. The simplest version allows the proportions to be read for each year (having been previously independently prepared as an input tape), or for the proportions to be generated by elementary formulae. For example, the value of the proportion p in the base year t is fed in with the trend slope in the proportion up to year $t+x_1$ followed by a second trend slope from year $t+x_1$ to year $t+x_2$. This sub-routine therefore allows either a polynominal or a polygonal function to represent the proportions as a function of time. Later, in more advanced applications, this sub-routine can be replaced by another of greater versatility and complexity; but it should be noted that there are some problems in the generation of complete sets of consistent $\sum_{s} (r,s)=1)$ proportions.

Figure 2

SIMPLIFIED FLOW DIAGRAM OF THE COMPUTER PROGRAMME


READ MODEL STRUCTURE AND RELATIONSHIPS
i.e., number of processes (boxes) with age groups (dimensions) and transition rules (origin list)
READ INITIAL POPULATIONS (STOCKS)
Base year values of all n(r, a, t)
PRINT ROUTINE AND ANALYSIS
Print resulting stocks in various forms with any analysis
IS THIS THE END OF THE CALCULATION ?
STOP
READ OR UPDATE TRANSITION PROPORTIONS
Read base year values of p(r, s, a, t) or generate values for year t + x and chech for consistency (i.e. $\sum_s p = 1$)
READ MIGRATION u(r, a, t)
AND CALCULATE FLOWS TO UPDATE STOCKS
$n(s, a + 1, t + 1) = \sum_r p(r, s, a, t)\, n(r, a, t) + u(s, a, t)$

The much more detailed model of the whole system used in our first large-scale calculations is described in the Appendix in the form of the specification required for the computer programme.

3. The data needs of the model

We have so far outlined only the foundations of our calculations. Before we build on this base, the data needs of the model must be discussed. This leads to consideration of how detailed the model structure can be, and raises questions of what data are already available, the prospects of filling the gaps in our knowledge, the possibility of making estimates to fill some gaps and, where all else fails, the adequacy of inventing data until they become available.

So far as "stocks" data are concerned, a considerable volume of data is now published. Even so, any ambitious plans to introduce more detailed models quickly run into gaps in our knowledge. For example, we do not know the age distribution of university students for each separate year of study. This can only be estimated by making certain assumptions about wastage and repetition rates for each age. Again, for certain purposes we might want to break down our teacher groups by subject taught, but these statistics are not known. However, though the required information is not directly known, it may be satisfactory to use statistics of teachers analysed into their main subject of qualification, which are known.

The fact that 'Statistics of Education' (3), with its greatly extended volume of data, has been published only for four years means that the time series for many of the details of interest are very short. Where they do exist, such time series may not be strictly acceptable due to changing definitions. For example, various bodies have used different degree subject classifications in the past, and an approved scheme of classification is a matter of recent agreement. There are changes in the actual structure of the educational system which have profound and diffuse repercussions on stocks. For example, starting in 1960, courses in teacher training colleges were extended from two to three years. It is virtually impossible to isolate the effects of such a change, or estimate what would have happenened had the change not taken place.

The availability of flow data is much more discouraging. The most reliable data relate to teachers because of the existence of the central record of teachers (an individualised data system set up for superannuation purposes). Until recently little attention has been devoted to flow data, and so educational returns have in the past concentrated upon statements of stocks at some particular time with no analysis of these stocks by the sources from which they immediately came. There are some isolated cases of flow data, but they remain few. For example, the school leavers survey gives information on the destinations of leavers (to university, further education, colleges of education and employment).

The volume of flow data can be expected to increase in the future and in the long run there is the prospect of an individualised data system. Any great mass of information collected and preserved will, in principle, contain much that we would like to know, but it will never be complete, since we are unable to anticipate all the information which may be relevant to future problems. Furthermore, the cost of extracting desired information could prove prohibitive so that an ID system cannot be blandly advocated for the model-building millenium. We will have more to say on this topic later.

For the present, then, where flow figures are lacking, we will have to estimate them, infer them from stock data or even invent them. In many cases, we have to resort to arbitrary procedures. Let us take the simple model whose movements are summarised in Table 1. Whenever a box is reached from only one source or leads to only one destination, this movement is determined. There is one movement determined in this way since box 1 can only be reached from itself (births being an exogenous supply which we take into account first). To proceed further, it is necessary to invent a priority rule for dealing with the remaining 17 movements. One possible rule is to allow as many people as possible to stay in the same process, i.e.

$$f(r,s,a,t) = \text{Min}\ [n'(r,a,t),\ n'(s,a+1,\ t+1)]$$

where the primes indicate the stock in box r which still remain to be distributed and the stocks in box s in year (t+1) which have still to be found after the movements which have so far been accounted for. This rule makes some appeal to common sense in so far as, say, a fourteen year-old boy in secondary school is most likely to have been a thirteen year-old in secondary school a year earlier. This rule would mean fixing the five movements: 1 to 1, 2 to 2, 3 to 3, 4 to 4, and 5 to 5 (not 6 to 6 since box 6 is partly a balancing item). Having done this, reference to Table 1 shows that the following movements are now determined as a result of applying this rule: 1 to 2, and 5 to 6. At the next stage, let us allocate 2 to 3 then 2 to 4, and 3 to 5 then 4 to 5. This determines 6 to 3, 6 to 4, 2 to 6, 3 to 6, 4 to 6, 6 to 5 and 6 to 6, i.e. the rest of the movements.

It must be stressed again that such procedures are arbitrary and alternatives could equally well be adopted. There can be no objective test of the results which can only be assessed in the light of what one intuitively feels is happening in reality. It will often happen that we will allow movements to occur in principle but that the flows determined by an arbitrary procedure will make them zero. Inferred flows are a temporary necessity but a poor substitute for data on actual flows.

It is worth noting that the problem arises because of the multiplicity of origins (or destinations) for a particular box. If we could re-define the transition proportions so that sources were unique the problem would be avoided. This could be done by defining the proportion in box r at time t in terms of the annual birth group from which they are drawn, i.e.

$$q(r,a,t) = n(r,a,t)/b(t-a-1)$$

where $b(t-a-1)$ is the number of births in the period $(t-a-1,t-a)$. It will be realised that this definition no longer represents an annual movement and pays no attention to the diverse paths which a student can take through the system and still reach the same point at a given time. It utilises the fact that the one thing that members of a model cell, $n(r,a,t)$, have in common is that they were born in the same year, but it neglects all details of the mechanics of the system. Nonetheless a model of this kind may be adequate for some purposes.

This alternative definition of movements makes us realise that our original definition of the transition proportions, p(r,s,t), takes account of the last step only (from box r) in the path through the system. There is a fundamental assumption here that the stock in box r in year t, n (r,t), is a homogeneous group; i.e. that all members of the box are equally likely to reach box s in year (t+1). Suppose that we have reason to believe that the performance of students at universities differs according to their secondary school backgrounds, and that this appears relevant to the purposes for which a particular model calculation is being carried out; then this would force us to introduce more detail into the classification embodied in the model structure. It would be necessary to distinguish, say, university science undergraduates who had come from one type of secondary school in one box, and those who had come from another type of secondary school in another box, in order to keep the boxes homogeneous. While our model language could cater for this kind of elaboration, it must be emphasised that full flow information is needed on each permitted movement. If we introduce further break-downs in our structure, but have to infer most of the flow data required by the new break-down, then the resulting exercises may do little more than display our imaginative ability.

Even in these early days of model-building it is apparent that the shortage of stock and particularly flow data will have an inhibiting effect upon our activities for some time to come. It may be the first major virtue of the model that it helps to clarify the future growth of data collection. The fact that there are many deficiencies in existing data makes our task more difficult, but it also makes us acutely conscious of the need for, and value of, an educational model.

4. Numerical example

For the purposes of illustrating a calculation with the computer programme, we are taking the sub-model of the school sector of education as specified in Table 3. This simple model is sufficient to provide two of the projections at present published in 'Statistics of

Education' - the projections of the school population and school leavers analyses by performance in school leaving examinations. It is also useful in making assessments of the effects of raising the minimum school leaving age from 15 to 16 in 1970-1. A simple extension of the model to sub-divide sixth forms according to specialisation in arts and sciences would permit the study of another matter of current interest, the apparently more rapid growth of arts courses in sixth forms than of science courses.

Table 3 describes the system for boys only. The structure for girls is the same, and Table 3 could be repeated with 14 added to each box number (including those in the origin list) to give the total system for both sexes. When this is done, the model consists of 28 boxes with 286 cells and 940 transition proportions.

The calculations were based on "stock" data for 1963 and 1964 as presented in "Statistics of Education". Flow data were not available and flows had to be inferred from these stocks using 1963 as the base year.

For a projection up to 25 years ahead, the results provide a considerable mass of material as printed out from the computer. For the present purposes of illustration it is sufficient to limit ourselves to some figures at five-yearly intervals and to confine ourselves to a comparison with the projection of the school population published in "Statistics of Education".

In Table 4 below there is a comparison of the last published projection and the results of a model calculation in which the transition proportions inferred from 1963-4 data are presumed fixed and apply throughout the period.

TABLE 3 Structure of model used in numerical example

Box number	Process	First age in Box	No. of ages in Box	Origin list i.e. this box can be reached from the boxes numbered
1	Primary............	2	14	1,14
2	Special............	2	18	1,2,14
	Secondary Schools Below "Sixth Forms"(1)			
3	Modern............	8	12	1,3,14
4	Grammar...........	8	12	1,3,4,14
5	Comprehensive.....	8	12	1,5,14
6	Technical.........	8	12	1,6,14
7	Other.............	8	12	1,7,14
	Sixth Forms of Secondary Schools(1)			
8	Modern............	15	5	3,8
9	Grammar...........	15	5	4,9
10	Comprehensive.....	15	5	5,10
11	Technical.........	15	5	6,11
12	Other.............	15	5	7,12
	Outside World (2)			
13	School leavers....	15	6	3,4,5,6,7,8,9,10,11,12
14	Outside world (3).	1	20	1,2,3,4,5,6,7,13,14

(1) "Sixth forms" of secondary schools provide courses for students in their 6th, 7th (and sometimes 8th) years in the school.

(2) "School leavers" consists of individuals in the outside world who were in school a year earlier; it is isolated from the rest of the outside world so that it may be seen separately as a projection. Box 14 therefore constitutes all people who are outside the educational system and did not leave school during the previous year.

(3) Flows are not known and have to be inferred from stocks in successive years. This leads to some flows to and from Box 14 (which is used as a balancing item) which are spurious. For example, movement from Box 4 to Box 14 should not be necessary as students leaving Box 4 should either go to the sixth form or become school leavers. These flows, which are small, are necessary because migration has not been introduced.

Table 4

Comparison of published projection with model calculation using fixed transition proportions

Projection A is published in "Statistics of Education", 1964, Part 2, Table 3.

Projection B is a model calculation using 1963 as the base year and fixed transition proportions from inferred flows between 1963 and 1964.

Boys and girls (in thousands)

Age	1968		1973		1978		1983		1988	
	A	B	A	B	A	B	A	B	A	B
2-4	259.4	255.6	282.5	275.9	300.9	288.2	313.6	292.2	336.4	308.9
5-10	4508.5	4498.3	5125.0	5099.3	5463.0	5438.7	5647.9	5620.9	5779.2	5750.2
11-14	2610.3	2631.6	2983.8	3016.1	3408.8	3435.6	3628.6	3657.8	3754.4	3780.4
15	425.4	405.1	694.0	439.1	808.7	509.6	881.8	555.6	926.4	584.3
16	188.9	195.1	358.7	206.0	441.1	241.6	514.9	267.3	568.8	281.9
17	104.4	111.9	118.8	112.4	160.1	133.5	202.8	151.6	237.6	160.3
18	37.4	37.7	41.1	37.8	52.3	42.7	68.7	49.8	81.7	53.1
19	3.8	4.4	4.0	4.4	5.0	4.9	6.6	5.6	8.0	6.1
Total	8138.1	8139.3	9607.9	9190.9	10639.9	10094.8	11264.9	10600.9	11692.5	10925.0

On first inspection it would seem that there is reasonable agreement between the two projections for 1968, but after this there is a widening gap with the model calculation steadily falling further below the published projection. There are two principal reasons for this divergence:

1. The published projection has taken into account the proposed raising of the school leaving age in 1971; and
2. Unlike the fixed transition proportions of the model calculation, the published projection makes allowance for recent trends towards staying on longer at school.

In order to take account of the proposed raising of the school leaving age in model calculations it is necessary to make some assumptions about the future behaviour of students who will be forced to stay on longer at school as a consequence of this decision. There is no evidence to indicate what is likely to happen but we can make assumptions which prescribe 'limits' upon what may happen. We may assume:

(i) The students who are 'forced', by the raising of the school leaving age, to stay on when they would otherwise have left will now take the characteristics of the people who stay on voluntarily at present, i.e. their future movements will be determined by the same transition proportions; or

(ii) The students who are forced to stay on will leave at the first legal opportunity.

These two assumptions may be regarded as the upper (i) and lower (ii) limits of behaviour. (This is not, of course, strictly true as the decision may also affect the behaviour of those who would have stayed on at school anyway; e.g. the decision may make it desirable for this group to stay on longer in order to maintain their "education differential" over the group who have been forced to stay on longer.) Under the second of these assumptions, (ii), students aged 15 who will be compulsorily kept in school will behave as under (i), but for those aged 16 and over, the numbers will be as in our first model calculation (B in Table 4). The comparison is set out in Table 5 below.

Table 5

Comparison of published projection with model calculation using fixed proportions (as in Table 4) combined with two assumptions upon the effect of raising the school leaving age in 1971

Projection A :	as in Table 4
Projection C :	fixed transition proportions, school leaving age raised in 1971, upper limit assumption (i).
Projection D :	fixed transition proportions, school leaving age raised in 1971, lower limit assumption (ii).

Boys and girls (in thousands)

Age	1973			1978			1983			1988		
	A	C	D	A	C	D	A	C	D	A	C	D
15	694.0	699.2	699.2	808.7	811.0	811.0	881.8	884.2	884.2	926.4	929.8	929.8
16	358.7	431.0	275.5	441.1	504.4	322.7	514.9	558.1	357.1	568.8	588.5	376.5
17	118.8	140.9	112.4	160.1	167.2	133.5	202.8	189.8	151.6	237.6	200.7	160.3
18	41.1	40.1	37.8	52.3	51.6	42.7	68.7	60.2	49.8	81.7	64.2	53.1
19	4.0	4.4	4.4	5.0	5.9	4.9	6.6	6.8	5.6	8.0	7.4	6.1

In general the published projection (A) falls between our two limiting projections (C and D). As time goes on, and particularly for the older age groups, the published projection tends to approach and exceed our upper limit. This brings out the importance in the published projections of the trends in transition proportions relative to the assumed effects of raising the school leaving age.

The introduction of trends in the transition proportions into our calculations depends upon the examination of a considerable mass of data and, at present, flows are being inferred for all relevant stock data since 1960. At this stage interim calculations have been made which take some account of the increasing tendency to stay on at school. To illustrate the possible effects of changing patterns of behaviour, linear trends have been introduced in a few of the transition proportions. All forward movements from boxes 3 to 7 for 14, 15 and 16 year-olds were examined for the years 1960 to 1964, and in most cases no detectable trends were observed. Token trends were introduced into only 11 of the 470 transition proportions, and half the changes were compensations, i.e. when the proportion staying on in box 3 at age 16 was increased by x per year, a compensating reduction had to be made in the proportion aged 16 leaving box 3 for box 13. The results of this calculation (for boys only) are set out in columns E (upper limit) and F (lower limit) of Table 6 so that they can be compared with the last published projection and previous calculations.

Table 6 shows the published calculation for the most part falling between the two limits E and F, though for 17 and 18 year-olds in the more distant future years, the published projection lies above the upper limit E. This is largely due to the fact that the published figures are the result of a different method of calculation which has incorporated a comprehensive treatment of trends.

It is necessary to stress again that this numerical example has been developed purely for purposes of illustration and the most important point which it demonstrates is the importance of the assumptions upon which projections are made. There is no evidence on the future behaviour of students who would not be expected to stay at school if the leaving age were not raised, and therefore no reason why a decision maker should choose either our "upper" or "lower" limit assumptions or any other assumptions. This raises the question of whether

TABLE 6

The effect of raising the school leaving age and trends in some transition proportions

Projection E: some trends in transition proportions, school leaving age raised in 1971, upper limit assumption (i)

Projection F: as E but lower limit assumption (ii).

Age	1983						1988					
	A	B	C	D	E	F	A	B	C	D	E	F
15.......	451.1	287.0	454.0	454.0	454.0	454.0	473.8	302.0	477.8	477.8	477.8	477.8
16.......	269.6	142.8	289.9	187.5	293.6	198.9	298.6	150.7	305.9	197.2	310.9	210.5
17.......	119.7	85.9	105.7	85.9	120.0	100.5	140.5	90.9	118.8	90.9	132.5	109.3
18.......	44.6	31.8	38.1	31.8	42.2	36.4	52.9	34.0	40.7	34.0	46.7	39.8
19.......	5.3	4.1	5.1	4.1	5.8	5.0	6.4	4.5	5.5	4.5	6.7	5.6

Age	1973						1978 Boys (in thousands)					
	A	B	C	D	E	F	A	B	C	D	E	F
15	354.6	227.5	359.8	359.8	359.8	359.8	412.7	263.2	416.3	416.3	416.3	416.3
16	186.2	109.8	223.6	144.4	225.0	152.4	230.0	129.2	262.4	160.7	264.9	179.5
17	69.5	63.5	78.3	63.5	81.3	70.2	94.3	75.8	93.2	75.8	101.3	86.1
18	26.9	24.1	25.8	24.1	27.8	26.3	34.1	27.3	32.7	27.3	35.0	30.5
19	3.2	3.2	3.2	3.2	3.6	3.6	4.0	3.6	4.5	3.6	4.8	4.2

a broad range of results of this kind provide a satisfactory basis for making decisions, and leads to a discussion of the limitations of these calculations and the purposes of model building.

5. Limitations

The basis for model calculations so far described in this paper is highly limited and we must now turn to the ways in which we wish to develop it.

Let us adopt a general viewpoint. If we possess stock data at yearly intervals only, it is still possible to make elementary calculations. The situation is similar to that of the 'fruit machine' or 'one armed bandit' whose windows can be read at the end of each shot only. Although nothing is known about what is going on within the system, 'black box' calculations are possible because future values of stocks can be forecast, however crudely, merely by extrapolating an empirical relationship based upon successive values in the past. The next stage is to introduce relationships or connections between the variables whose values are measured annually. This corresponds to the introduction of channels into the black box, and the flows, $f(r,s,t)$, are the traffic down these channels.

This is the limit of the calculations described so far. If the transition proportions are changing from year to year, then the updating is achieved by extrapolating empirical relationships. However, as soon as we begin to talk about 'the demand for education', we postulate that flows depend upon such factors as social class, school and home environment, etc. In effect we are beginning to suggest mechanisms that underlie the demand for education and, in principle, these mechanisms should be built into our procedures for changing transition proportions. This is not, however, a simple matter. Firstly, the dependent variable may not be easily measured and may have to be represented by a related 'proxy' variable, e.g. parental income for social class. Secondly, the introduction of variables of this kind may lead to the fragmentation of the model into many more parts, i.e. boxes and cells as such variables may pervade the whole system. This does not rule out all demand mechanisms. For example, it has been suggested, with respect to the switch towards the arts in the upper grades of schools, that the cream of science graduates may be attracted to industry. Since students are influenced by the calibre of their teachers, a decline in the numbers of first class science teachers could be self-aggravating as it would lead to fewer applicants for science places in universities and subsequently fewer science teachers. It is possible that this mechanism could be represented, at least crudely, by making the numbers of students moving to science specialisations depend partly upon the numbers of teachers with science qualifications. That is to say the flows could be partly controlled by the student-teacher 'gear' ratios for each subject, with the flow decreasing as the ratio increased.

Apart from demand mechanisms, we must also introduce supply mechanisms. So far it has been taken for granted that the flow, f(r,s,t), can be accepted by process s at time (t+1). In practice we know that at many points the demand for places in recent years (e.g. in universities) has been considerably greater than the supply of places. These checks upon demand mean that we must now introduce the notion of limited capacities to the channels or paths through the system. We may define the maximum number of people who may be in process s in year t as c(s,t) with the values of these limits determined by the decisions of educational administrators who take into account the demand for places and the possibilities of expansion, i.e. the availability of teachers, buildings, land and finance, and many other factors.

The introduction of bottlenecks (the situation where a flow may be limited by the channel capacity) is a major extension of the model language and presents numerous problems. Where a bottleneck is possible we can no longer write

$$n(s,\ t+1) = \sum_r p(r,s,t)n(r,t) + u(s,t)$$

but must modify this to equations of the kind

$$n(s,\ t+1) = \text{Min}\ \left\{ \left[\sum p(r,s,t)n(r,t) + u(s,t)\right], \left[c(s,\ t+1)\right] \right\} \qquad (2)$$

Furthermore, when a limit is applied it has repercussions upon the rest of the system. If we think of a real bottleneck, such as entry to university, we will realise that there are two consequences of a limit: (a) the educational authorities have to make use of selection procedures which may discriminate against some of the ways getting of box s; and (b) the students who do not gain entry to box s have to act in ways which redistribute them into other parts of the system. The preference procedures of frustrated students are capable of much greater complexity than the selection procedures of the authorities because the student may have several alternative courses open to him and these may be permutated into every possible priority order.

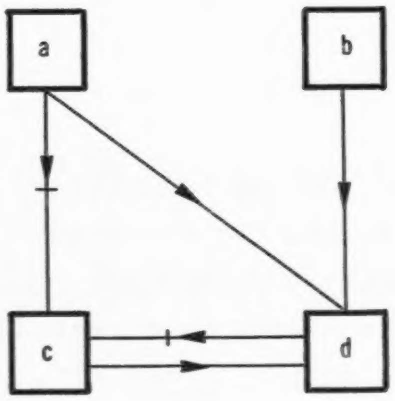
Suppose, for example, in the simple model of Table 1, that there was a shortage of university places, i.e. a bottleneck into box 3. Box 3 can be reached only from box 2 apart from itself and the "outside world", box 6. However, students from box 2 rejected for box 3 will be redistributed to their other possible destinations, boxes 2, 4 and 6. In this way the effect of the bottleneck spreads through the system. The rejected students have the alternatives of staying in box 2 (i.e. staying where they are, and probably hoping to apply to university again the following year), or moving to Colleges of Education or to the outside world. A proportion, p_1, of the rejected students may place these alternatives in the order 2,4,6; p_2 may place them in the order 2,6, (even when there are more alternatives an order may always be terminated at box 6 since we can take it that there is never any restriction upon entering the outside world); p_3 may have the priority order 4,2,6; p_4 may have 4,6; p_5 are neither prepared to wait at school nor to accept any educational alternative to university, so their choice is simply box 6. In this example any preference procedure can be specified by fixing p_1, p_2, p_3, p_4, and p_5. In general there are many alternatives and this may become cumbersome.

The bottleneck problem presents more difficulties. So far we have spoken of only one bottleneck but there may be many and they may be related. The overspill from one bottleneck may be sufficient to cause a bottleneck at another point where it would not otherwise have occurred. Single bottlenecks ((i) in Figure 3), and unconnected multiple bottlenecks (ii), could be handled quite easily but we must expect to find connected multiple bottlenecks (iii) as the most common occurence.

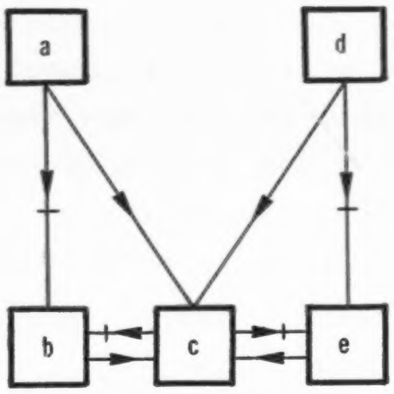
Figure 3

EXAMPLES OF TYPES OF BOTTLENECKS

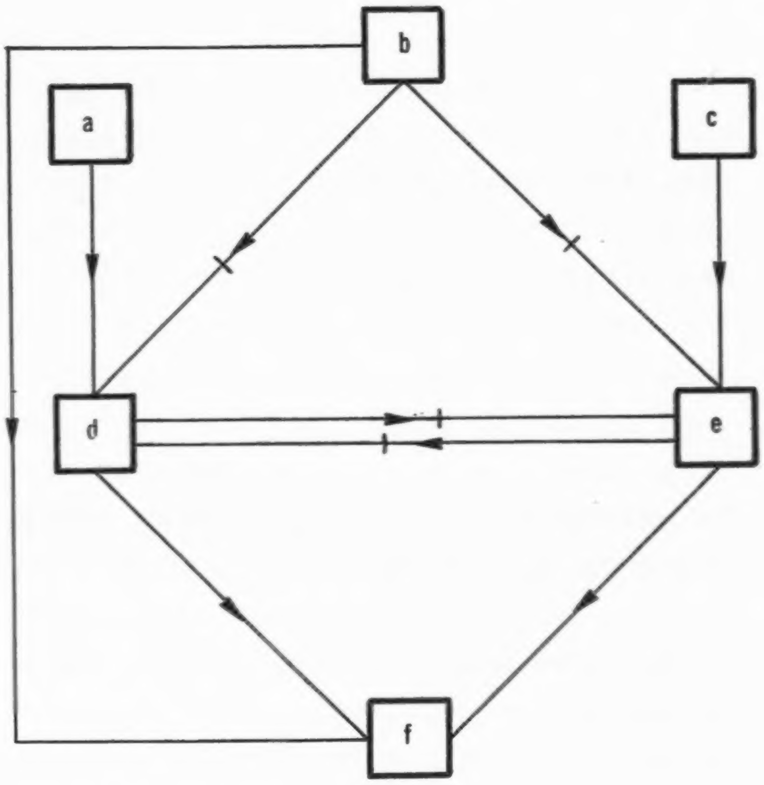
(*Note:* A bottleneck is indicated by a bar drawn across some or all flows into a particular box)



(i) **Single**



(ii) **Multiple, unconnected**



(iii) **Multiple, connected**

The modification of the equations to allow for bottlenecks means that, when we have calculated all values of $n(s,t+1)$ according to (2), it will be necessary to check whether any limits have been exceeded and, if they have, to apply the selection procedures and then redistribute the excesses by further iteration using second preferences. Iteration can be avoided if we are prepared to place our possible bottlenecks in a priority order and work through th[illegible]s,t+1) in this order. These alternative procedures correspond [illegible] ifferent real situations. For example, if we make the universities our first bottleneck then under a priority system, frustrated university applicants could have an equal chance of successful application to Colleges of Education with students who made this their first choice of action. Under an iterative procedure frustrated university applicants might find that all or most College of Education places had been allocated at the first iteration, though they could have confidently expected to gain a place if they had made this their first application. A general treatment of the educational model with multiple bottlenecks would allow great versatility in the selection and preference procedures. This will be the subject of a later paper.

The introduction of bottlenecks has a further implication for the nature of transition proportions. According to our previous definition, the value of p(r,s,t) expresses the number moving from r in year t to s in year (t+1) as a proportion of those in r in year t. The p(r,s,t) so defined represent the flows which result after all the conflicts in the system have been resolved. However, if we introduce bottlenecks into the model, we are interested in the numbers who want to go to s in year (t+1). Clearly parameters, p(r,s,t), expressing these numbers as proportions of those in box r in year t cannot be estimated by looking at past data, as this cannot tell us how many students were frustrated by bottlenecks. This point has implications for the future development of data collection, since it implies a shift from incorporating purely empirical relationships to embodying mechanisms or motivations in our models. For the present it may be highly instructive to assess how sensitive the future development of the system is to alterations of demand proportions.

Even at this stage, our model calculations will still be purely deterministic, i.e. everything happens exactly as speciified by the equations. We have avoided describing the transition proportions as probabilities because it is arguable that it is wrong to think in terms of a member of box r as having the chance p(r,s,t) of going to s at (t+1), and wrong to think that values of p(r,s,t) over successive years represent repetitions of the same probability trial situation. If we are to make some assessment of the fact that things hardly ever happen exactly as specified and some assessment of the value of deterministic calculations, then it is convenient to regard the transition proportions as probabilities. The proportion, p(r,s,t), can then be regarded as having been drawn from a multinomial distribution since $\sum_{s} p(r,s,t) = 1$. The model will then consist of simultaneous multinomial distributions and while the explicit treatment of such a system would be intractable, it will be possible to carry out Monte Carlo simulation calculations in which the values of all f(r,s,t) are found by sampling from multinomial distributions. The introduction of random variation in this way is one line of attack upon the 'noise' problem, i.e. the distortion of the signal of the deterministic calculation by the presence of variation, the fact that things do not happen exactly as expected. The deterministic calculation can still be regarded in the traditional way as providing 'the best estimate' of what is expected to happen; but these simulation calculations will give some indication of how far we are justified in placing our faith in these 'best estimates' and acting upon them.

6. Discussion

We have proceeded so far on the presumption that the basic language and the forms of calculation to be made will be similar no matter how the use of models is developed. A discussion of the purposes and scope of educational model building can be delayed no longer.

The need to provide projections for such committees as Robbins and the National Advisory Council on the demand and supply of teachers was an important stimulus towards the development of a model. In considering such problems as the growth of higher education and teacher supply, these committees required information on the likely development of various parts of the system. The projections provided forecasts of how the system could be expected to develop if the components considered continued to behave as in the recent past, i.e. by extrapolating recent historical trends, or if certain policy decisions were taken to alter these trends. It is difficult to know precisely how the committees used these forecasts to reach their conclusions, but it does not seem unreasonable to say that they formed only one piece of evidence in their deliberations and that the forecasts were only loosely connected or even independent of other issues germane to the particular problem. It is not our intention to suggest that more weight should have been placed on the forecasts (indeed the opposite may be true), but that the projections were only one element in their deliberations and that this method of procedure represents the most elementary use of model computations.

The next stage after forecasting is to consider the model as a tool in educational planning. We must take great care with this term since it means many things to different people. For the purposes of an article on 'Educational Planning', Diez-Hochleitner (4) defines a plan to include 'the overall objective of education', whereas a programme is 'a more detailed determination of specific objectives to be achieved in a specific time schedule'. By 'planning', we mean an activity which embraces both the drawing up of plans and the study of possible programmes to realise them.

At present, it could be argued, a plan may emerge out of, or is implicit in, the decisions of the educational administrators who are charged with operating the system. They have to take specific decisions: when to raise the school-leaving age, when to extend teacher training courses, when new schools should be built, how to produce more teachers, and so on. While efforts are made to anticipate the ramifications of decisions, it would appear that decisions are often taken individually and that the procedure is piecemeal. To an outsider a group of decisions may seem inconsistent, incompatible and exactly contrary to needs and avowed aims. For example, a journalist can write:

> "This is the way British education works, by faith and brave lurches in the dark. In times of desperate teacher shortage, we raise the school leaving age and extend the teacher training course. Then, overhelmed by troubles, we feud over whether untrained auxiliaries should be allowed into class-rooms, whether the training courses should be boxed up. It is a curious sort of progress, without a broader view, without a true perspective of change." 5)

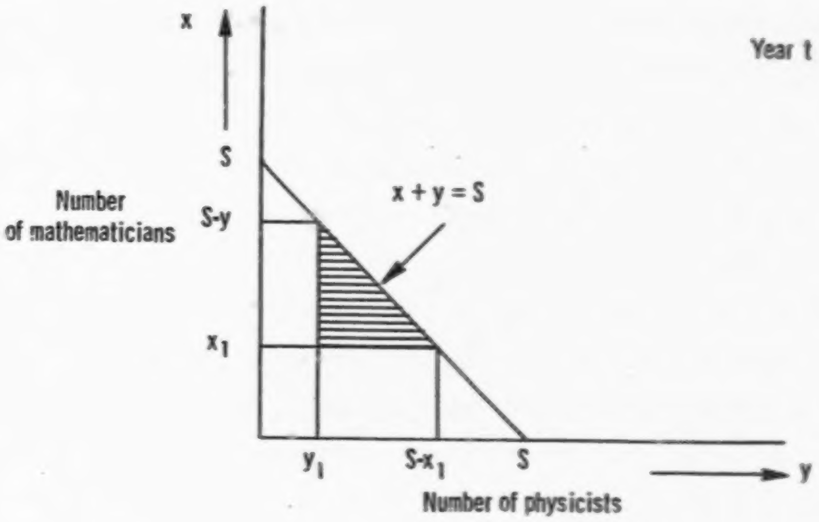
The 'broader view' is usually adopted by economists, social scientists, politicians, and particularly those who are concerned about the 'output' of the educational system. They may wish to map out desired future states of the system in order to discuss how these might be achieved. The first important feature of this approach is that it sets targets. It matters little whether these targets are the result of complex projections of the labour force converted into educational requirements or whether they are simply works of imagination. Even if the approach is carried no further, these targets may have a psychological value insofar as they may produce a response if sufficiently publicised. However, it is logical to go further and ask how these can be achieved, and, because there may be many ways of reaching them, which is the best method of achieving them. There may be great difficulty in defining acceptable objective criteria for choosing one way rather than another but this is the second important concept introduced by this approach. Before we go any further we must clarify what we mean by targets and objective criteria.

To say 'We want x mathematicians in future year t', is not sufficient to specify a target. A policy which leads to any number of mathematicians greater than x in year t will not necessarily be satisfactory because we may have produced mathematicians at the expense of physicists who have fallen below some desired minimum number, y, of physicists in year t. Consequently a target constitutes a complete specification of the mathematicians, physicists, and every other skill included in the model (i.e. the state vector). Furthermore, it is misleading to think of the target as a point to be aimed at like a bull's eye. A target is an acceptable region defined by a set of compatible constraints (e.g., $x \geqslant x_1$, $y \geqslant y_1$, $x + y \leqslant S$ ...).

This can be illustrated by a crude example. Suppose that the world consists only of artists and scientists and the latter must be either mathematicians or physicists. If there are s ($s \leqslant S$) scientists in year t, x of them are mathematicians and $y = (s - x)$ are physists. The situation is illustrated in Figure 4. Any policy which brings us into the shaded region is an acceptable policy because it provides us with at least x_1 mathematicians and y_1 physicists. Although this example is crude it provides some insight into the nature of a target, and it may in principle be generalised into n variables by considering an n-dimensional space.

It is also important to note that we are not interested only in reaching a target area in year t. It is not satisfactory to reach an acceptable region in 1973 but not be in an acceptable region in 1970, if this can be avoided. It is not satisfactory to be in the acceptable region in 1973 but so disposed that it is impossible to be in the (expected) acceptable region for 1976. Consequently, if

Figure 4



we introduce time as a third dimension into Figure 4, the acceptable region becomes a 'funnel' over time. This 'funnel' will widen into the future since there will be much less precision and certainly about distant targets and much more time to take corrective action to achieve them.

The definition of objective criteria by means of which the different ways of reaching a set of targets may be compared and the best path selected also presents difficulties. There will be many alternative criteria and ideally we would like to find some way of weighting the different criteria, i.e. compounding them into a unique objective function. This is usually done by introducing a cost function which must be minimised. This raises many questions. The solution must be constrained; for otherwise it would be cheapest not to provide any education at all. If constraints are added that education must meet the legal demands of compulsory education and that student-teacher ratios must not exceed specified values, then the minimum value of the cost function is likely to be a boundary value; i.e. we should not go beyond the minimum demands of the law or provide more generous student-teacher ratios than have been specified as the acceptable limit.

Such a cost function would not be acceptable as an operational guide. There are many further difficulties. Should costs borne both nationally and locally be taken into account ? Should indirect costs be taken into account? If so, should penalty costs be assessed when the system fails to produce a desired output? What is the cost of failing to produce x mathematicians in year t? Over what time period should costs be minimised and how should the future be discounted?

These questions are sufficient to suggest the difficulties of defining an objective function. We may not be able to develop one such function, but we may be able to suggest several separate criteria which must be satisfied simultaneously. In practice, one crude constraint might be that the costs of education must not exceed x% of the gross national product. If the system falls outside an acceptable target region at any particular time, our concern may be to get it back into an acceptable state as quickly as possible. This suggests a general criterion which takes into account the manoeuvrability of the system, i.e. a criterion which allows us to choose the course of action which maximises the possibilities for corrective action taking into account at each point of time the feasible range of developments.

These attempts to clarify what we mean by targets and objective criteria make it apparent that the process of planning, in terms of deciding what is wanted and judging the best way of getting there, is not as simple as it appears. This conception of planning has other shortcomings. The proposed procedure seems to be that exercises will be carried out at regular intervals to establish targets and policies. There is no indication that the lessons of one plan would affect subsequent plans. Unless reviews and revisions were very frequent, there is a possibility of appreciable time-lags between things going wrong and their consequence upon new plans being considered. There is also the risk that planning will be only loosely related to the actual day-to-day processes of decision making. Equally important, this kind of planning, like simple forecasting, has to take for granted the reliability of projections over the period considered. If projections are very unreliable, such planning is completely undermined. And, while educational projections have a short history, the evidence from related fields, like demography, does not inspire much confidence. It can also be argued that it is inadvisable to [illegible]n many years ahead when decisions have been taken, and are likely to be taken, which will produce qualitative and structural changes in the near future whose impact on the medium and longer term cannot be predicted.

A model of the whole educational system will provide a more coherent and consistent background for clearly making decisions, but it may provide much more. Professor Stone has written of his work on a model of the economy:

> "In these models, the government of a modern economy in which public and private enterprises are mixed is thought of as the classicalsteersman of the ship. Certain things are under its direct control; it has its instruments of policy which it can manipulate directly; part of the crew it can give orders to, others it must persuade; and surrounding the ship is the sea of economic events which obeys natural laws of motion beyond the steersman's influence. The model then produces lists of rules for the steersman to follow if he wants the ship to hold a certain course. We on the other hand, start from the point of view that the responsability for decisions which affect the form and growth of the economy is, in fact, widely dispersed. These decisions are made in central and local government, in nationalised and private industry, by workers, managers and consumers. Our purpose is to provide the means whereby all these decisions can be taken against a more objective background of consistent information than is commonly the case" (6)

While this may be true for the economy, the educational system is not quite so complex, and its organisation should be far more responsive to public command. Certainly it is worth investigating whether the educational model can be more intimately related to the process of making decisions. In a review of Professor Stone's work, Dr. Jeremy Bray makes the important point that there are dangers in providing background information which is then used, somewhat mysteriously, to reach actual decisions:

> "... Before going any further it would be wise to stop and ask precisely how such a model system should be used, who should use it, and how it could be made easy for them to use. It is easy for the model builders to say that it is not their job, which is just to provide the model for others to use. It is equal-

ly easy for those in government and industry faced with decisions to say that they cannot use the model because it does not deal with the practical questions with which they are faced." (7)

It is implicit in the argument so far that we are attempting to control or govern the educational system as a continuous on-going process. At the present time adaptive control theory is being intensively developed with many successful applications in industry and space research, and it is informative to see what this approach has to offer.

The problems of adaptive control are described in similar terms to those we have used. The system has an output which is controlled by manipulating the inputs. This manipulation aims to bring out as close as possible to a target, the 'distance' being measured by some performance index of the system. The present developments in adaptive control have sprung from 'an acceptance of the necessity to deal with non-linearity' and the realisation that systems are put under control because they are subject to vagaries of performance arising from random disturbances or parameter variations'. (8) A dynamic control strategy works out how to alter the available control variables by the application of a minimization principle. It is adaptive insofar as it takes account of the evolution both of the system and of our knowledge of it. By utilising fresh information, parameter values are revised and control made more effective.

Adaptive control procedures do not appear to have been applied so far in socio-economic contexts. The principal reason is that the procedures are best suited to well-defined situations. Although we have attempted to clarify the notion of an educational target above, we have not yet brought out its dynamic characteristics. In a chemical process the target output may be fixed for a long time ahead or it may change in a very well-behaved manner. Our educational targets are not so precise. Normally they will depend partly upon manpower forecasts which we have no reason to beleive are more reliable than educational forecasts. Since targets are not entirely determined by what is happening within the edicational system, and are subject to revision, we find that we have moving targets which are not very precise and subject to 'shock displacements' from time to time, e.g.

due to political decisions and political changes. By comparison, a moon-shot is well defined.

We have already made some comments on the difficulties of defining objective functions or performance indices which can be subjected to minimisation. Further difficulties arise from the fact that'manipulating inputs' to aim at desired results is also less well defined in socio-economic systems. In other fields where adaptive control has been successfully applied, there are means of manipulating corrections according to predetermined relationships and these may take the physical form of calibrated control knobs. While educational decisions are taken and acted upon in the belief that they will have some effect, the range of possible actions and their effects are not so well understood that it could be described as a 'steering mechanism' with which to control the system. However, it is useful to think in these terms, to study the nature of educational decisions and to invent possible control 'knobs'. This raises many questions because, in our system, powers of central direction are highly limited, and policies are largely aimed at influencing the decisions of the educational authorities and teachers on the one hand and the students on the other. How do you control decisions to build new schools? How do you attract more graduates into teaching or any other profession? How do you persuade a graduate to teach in a specific area rather than give up teaching? How do you persuade students to follow a course different from that which intended first? How do you dissuade students from specialising in an (at least, temporarily) 'unwanted' subject?

It is not difficult to suggest answers to each of these questions, but it is difficult to coordinate and quantify the possible answers so that they represent a coherent means of changing the direction of the whole system from the course upon which it seems to be set. Suppose, for example, that special grants and funds are available in order to persuade people to follow desired courses of action.
This could either take the form of (a) funds for selected institutions or faculties; or (b) preferential grants to sixth form students who pursue desired courses; or (c) more pay for teachers. The effects of these 'knobs' can be theoretically argued, but they cannot be predicted

with any accuracy. It may even be argued that proposed policies will not have an effect in the desired direction. For example: (a) it is pointless to make more places available if there are not the teachers to support them; (b) preferential grants to sixth formers may aggravate the situation when lack of places is the dominant factor; (c) raising teachers' pay increases educational costs and this might have little immediate effect if industry, competing for graduates, can react to future shortage very rapidly.

It is clear that there is difficulty not only in defining the control knobs of the educational system but also in calibrating them so that our control actions constitute rational strategies.

We may be able to argue the direction of an effect without being able to anticipate it more precisely; e.g. increasing teachers pay should lead to more teachers, but it is quite impossible to say that an increase of £y per annum should lead to x more teachers in year t. Even where past data exists to throw some light upon such a point, it will not usually be sufficient since it will relate to a different environment, i.e. in considering the effect of a teachers' pay rise we should take into account the competitive industrial salaries when this was last done and their values now. New policies will frequently be advocated for which there is no historical precedent, and in this case calibration is impossible except by pure speculation.

It may also be a problem to invent 'infinitely variable knobs' in socio-economic systems. It may be easy to invent knobs which can reasonably be expected to boost some part of the system, but very difficult to invent knobs which reverse or damp down an undesired trend. For example, grants may be introduced at some propitious time, but it may be difficult to discontinue them when the need is not so keenly felt as this could become a political issue. Again, it is always possible to increase teachers' pay but it is unlikely that it could ever be reduced as a means, say, of discouraging graduates from wanting to teach particular subjects. The importance of devising policies which can reverse trends is apparent from the potential uncontrollability of a system corrected only by 'one-way knobs' (cf. the drug addict increasingly dependent upon stimulants).

The essence of adaptive control is that the idea of learning more about the system from experience is built into the continuous process of control. It can be argued that the present methods of data collection also lead to better understanding of the system, but they are not designed from the viewpoint of decision making. The philosophy behind existing data collection would seem to be historical rather than operational, successfully aimed at generating annual descriptions of the system. In recent times there has been a shift of emphasis to include flow data as well as stock data and these improvements enhance the precision and usefulness of the statistics. However, their practical value is mainly explored in an intermittent fashion, for example when a surge of interest in a particular topic leads to the setting up of an ad hoc committee. The needs of decision-making do not have direct influence on the way the knowledge of the system is improved as they would if data collection was completely orientated towards controlling the system. The implication of control procedures is that the flow of information should be developed and focussed so that it is directly and comprehensively relevant to the operational guidance of the system. It is not being suggested that existing data collection should be diminished but that further collection should be comparable to the continuous monitoring that takes place in the control of industrial processes. In particular we will want to know much more rapidly what is happening, i.e. we will want to 'meter' the system at much earlier stages, even if this means resorting to information from small samples. For example, if a policy was directed to shifting the balance of arts and science entrants to universities, some information on the response to this policy would become available as soon as university applications were received, if not at an earlier point in the school system. Consequently, if a satisfactory response to this policy was critical and an unsatisfactory response would oblige further action, then it would be desirable to 'plug in' to a suitable point (in this case the University Central Council for Admissions to whom applications for entry to university are made) to get the earliest possible indicator rather than to wait for the summary of university admissions many months later when action would already be too late. Thus, the most immediate value of this approach to model building would be to focus attention on the key points at which data collecting effort should be concentrated.

7. Conclusion

Our review of the possible development and use of the computable model in educational planning has carried us some way from our starting point in forecasting. However, we point out that each of our stages - projection, target-planning and control - is the prerequisite for its successor. In order to aim at a target, we need to project the consequence of various alternative policies. In order to carry out a continuous control of the system, the degree of success in approching previous targets must be weighed, allowing for random and partially known influences.

Let us restate our present view of the purpose of educational model building in the following form.

1. The educational system is complex and dynamic. From day to day, in the light of the relevant, available information, decisions are taken with the aim of achieving specific objectives. The more explicit, rational and exact this process is made, the more the decision makers will be able to consider the consequences of their decisions for the entire system.

2. The starting point of this approach lies in the clarification of objectives. External demands are placed upon the system (e.g. manpower requirements) and past and present educational performance implies further targets (i.e. social demand). An attempt should therefore be made to combine and weight the numerous objectives of the system according to their relative importance in an objective function. Constraints must be associated with this function in order to specify the limits of acceptable solutions, so that policies which could not be applied in practice are excluded. The model of the system must comprise all the integrated elements which are believed to be significant and the parameters of the model must be determined from past data. The model can then be used to project the future development of the system under various decision patterns. The application of the objective function leads to the selection of the best sequence of decisions.

3. The functioning of the process may therefore be broken down into three stages. Firstly, the output of the educational system is compared with the output of the model. The parameters of the model are adjusted to obtain the 'best fit' of the model to the system on present and past data. Secondly, the model is used to project the future output of the system assuming various patterns for future decisions. The objective function enables us to select the set of decisions expected to produce the changes in the educational system which will best meet future targets. The third part is a monitoring check on the performance of the system after these decisions have been applied. If the discrepancy between the actual effect and the expected effect is too large, then further changes are called for in the form of corrective action. The objective function may be amended or the model structurally altered. The whole process is cyclical and continually operative.

4. The procedure described above follows control engineering in focussing attention on certain aspects of decision-making. It emphasises the interrelations and reciprocity between events, information, objectives, targets and decisions. Targets may be modified in the light of available information and the chosen sequence of decisions is determined as a reconciliation of the information fed back and feasible targets. Because information is incomplete and because the effects of decisions must be verified and, if necessary, corrected, there is a need for fresh information. The procedure reflects the on-going nature of the system. The model continually improves its representation of the system, and adapts to its changing structure and objectives.

5. Our approach seeks to avoid the danger of distorting the problems really confronting educational decision-makers by forcing them into prefabricated algebraic and computational straight jackets. No computable model can provide perfect predictions of the future or decide the objectives of the system.

Far from implying the elimination of experience and intuition the embedding of the model in an adaptive control system can be no more effective than the human judgments embodied in the objective function. Indeed, the model, viewed as a control instrument, endeavours to enhance the influence of the decision makers over the consequences of their decisions.

Appendix

DESCRIPTION OF FIRST LARGE-SCALE MODEL CALCULATIONS

The structure of the first large-scale model to be assembled for the computer is described below in the style in which it has to be drawn up to fit the computer program. The description (see Table I) applies to both sexes, and individual ages are used throughout. The data relates to England and Wales only.

The data tape required by the program must first state the total number of boxes; in this case 164, 82 for each sex. The following information for each box (in numerical order) is then required :

(i) First age in this box;

(ii) Number of cells (i.e. individual ages) in this box;

(iii) Number of origins for this box;

(iv) Box numbers of these origins;

(v) Base year values of cells in this box.

When this information has been given for every box, the transition proportions for the base year for all movements follow in the order determined by the sequence of origin lists, i.e. the first 13 numbers, the transition proportions for movement from box 1 to box 1 (i.e. one for each of 13 ages), are followed by the 14 transition proportions for box 81 to box 1, and so on. The total number of transition proportions is over 10,000 for each sex. For runs in which the transition proportions are up-dated each year, the values of parameters describing the changes, e.g. trend increments, must now follow.

The base year for the calculation is 1961 and most of the stock data required had been published and required only minor adjustments. Very few flow data were available and most of the transition proportions had to be inferred.

A more detailed description of the data problems and of the 'results' of computations will be given in a later paper.

Table I

Description of structure of first large-scale computer calculation

Box number	Sector	First age in box	No. of ages in box	Origin list, i.e. this box can be reached from the boxes numbered
	Primary			
1	Primary	2	14	1,81
2	Special	2	18	1,2,81
	Secondary			
3	Modern	8	12	1,3,81
4	Grammar	8	12	1,3,4,6,7,81
5	Comprehensive	8	12	1,3-7,81
6	Technical	8	12	1,6,81
7	Other	8	12	1,7,81
	University undergraduate			
8	Arts Year 1	17	7	3-7,8,35,81
9	2	18	7	8,9
10	3	19	9	9,10
11	Final(1)	19	9	9,10,11
12	Social studies Year 1	17	7	3-7,12,35,81
13	2	18	7	12,13
14	3	19	9	13,14
15	Final	19	9	13,14,15
16	Pure science Year 1	17	7	3-7,16,35,81
17	2	18	7	16,17

Box number	Sector	First age in box	No. of ages in box	Origin list, i.e. this box can be reached from the boxes numbered
18	Pure science Year 3	19	9	17,18
19	Final	19	9	17,18,19
20	Applied science Year 1	17	7	3-7,20,35,81
21	2	18	7	20,21
22	3	19	9	21,22
23	Final	19	9	21,22,23
24	Medicine Year 1	17	7	3-7,24,35,81
25	2	18	7	3-7,24,25,81
26	3	19	7	25,26
27	4	20	7	26,27
28	5	21	7	27,28
29	6	22	9	28,29
	University post-graduates			
30	Arts	19	22	11,30,40,79
31	Social studies	19	22	15,31,44,79
32	Pure science	19	22	19,32,48,79
33	Applied science	19	22	23,33,52,79
34	Medicine	19	22	29,34,56,79
	Further education (vocational colleges etc.)			
35	Non-qualification (2)	15	15	3-7,35,81
36	Non-advanced (3)	15	15	3-7,36,81
37	Arts Year 1	17	13	3-7,36,37,81
38	2	18	13	37,38

Box number	Sector	First age in box	No. of ages in box	Origin list, i.e. this box can be reached from the boxes numbered
39	Arts Year 3	19	13	38,39
40	4	20	15	39,40
41	Social Year 1	17	13	3-7,36,41,81
42	2	18	13	41,42
43	3	19	13	42,43
44	4	20	15	43,44
45	Pure science Year 1	17	13	3-7,36,45,81
46	2	18	13	45,46
47	3	19	13	46,47
48	4	20	15	47,48
49	Applied science Year 1	17	13	3-7,36,49,81
50	2	18	13	49,50
51	3	19	13	50,51
52	4	20	15	51,52
53	Medicine Year 1	17	13	3-7,36,53,81
54	2	18	13	53,54
55	3	19	13	54,55
56	4	20	15	55,56
	Teacher Training Institutions			
57	Post-graduates	20	50	11,15,19,23,40,44,48,52,57,79
58	One year course	18	52	3-7,35,36,58,81
59	Two year courses Year 1	18	52	3-7,35,36,59,81
60	2	19	51	59,60
61	Three year courses Year 1	18	52	3-7,35,36,61,81
62	2	19	51	61,62

Box number	Sector	First age in box	No. of ages in box	Origin list, i.e. this box can be reached from the boxes numbered
63	Three year courses Year 3	20	50	62,63
	Teachers			
64	Maintained primary schools Grads.	19	51	11,15,19,23,57,64,66,68,69,71,73,75,79
65	Non-grads.	19	51	3-7,36,40,44,48,52,58,60,63,65,67,68,70,72,74,76,80,81
66	Maintained secondary schools Grads.	19	51	11,15,19,23,30,31,32,33,57,64,66,68,69,71,73,75,79
67	Non-grads.	19	51	3-7,36,40,44,48,52,58,60,63,65,67,68,70,72,74,76,80,81
68	Maintained and direct grant special schools	19	51	57,58,60,63,64-67,68,79,80
69	Direct grant schools Grads.	19	51	11,15,19,23,30,31,32,33,57,64,66,68,69,71,73,75,79
70	Non-grads.	19	51	40,44,48,52,58,60,63,65,67,68,70,72,74,76,80,81
71	Independent schools Grads.	19	51	11,15,19,23,30,31,32,33,57,64,66,68,69,71,73,75,79

Box number	Sector	First age in box	No. of ages in box	Origin list, i.e. this box can be reached from the boxes numbered
72	Non-grads.	19	51	40,44,48,52,58,60,63,65,67,68,70,72,74,76,80,81
73	Teacher training colleges Grads.	19	51	64,66,68,69,71,73,75,79
74	Non-grads.	19	51	65,67,68,70,72,74,76,80
75	Further education colleges Grads.	19	51	11,15,19,23,30,31,32,33,57,64,66,69,71,73,75,77,78,79
76	Non-grads.	19	51	36,40,44,48,52,58,60,63,65,67,70,72,74,78,80,81
77	Colleges of advanced technology	19	51	30,31,32,33,66,69,71,75,77,78,79
78	University	19	51	30,31,32,33,66,69,71,73,75,77,78,79
	Outside World			
79	Graduates	19	51	11,15,19,23,29,30-34,40,44,48,52,56,57,64,66,68,69,71,73,75,77,78,79
80	Ex-teachers (non-graduates)	19	51	58,60,63,65,67,68,70,72,74,76,80

Box number	Sector	First age in box	No. of ages in box	Origin list, i.e. this box can be reached from the box boxes numbered
81	Other Outside World	2	68	1-63,81
82	Deaths	2	68	1-81

Notes :

(1) "Final" means year in which final examination must be taken and this will be the third or fourth year. Consequently "year 3" represents students in third year or later not taking final examinations.

(2) "Non-qualification" covers courses not leading to a recognised qualification.

(3) "Non-advanced" covers courses leading to a recognised qualification but these are not advanced as are the courses covered by boxes 37 to 56.

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PART IV : OPTIMIZATION

GENERAL OPTIMIZATION MODEL FOR THE ECONOMY AND EDUCATION

by Jean Benard

I. GENERAL CHARACTERISTICS

1. Purpose and general structure of the model

The model described below is set out in the form of a sequential linear programme embracing the entire national economy, education being treated as one of the "sectors" or "activities" of the economy. Its purpose is to determine the optimum allocation of resources, mainly physical, between education (considered as an activity), and the "commercial" economic activities represented by the sectors of an

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(1) I take this opportunity of thanking the members of the "Collective Requirements" team of my staff at C.E.P.R.E.L.: Mrs Girardeau, Miss Rouard, Mr. Ettori, Mr. Manuel and Mr. Terny, who helped me to develop this model and are now evaluating its parameters and working it out.

inter-industry input-output table. In this system, the educational sector is regarded as a producer of the knowledge needed for the training of skilled workers employed in all sectors. This optimised allocation is obtained by maximising, subject to constraints, a social preference function which is itself made up of numerical indices of the standard of living of the population throughout the years considered and of the production potential at the end of that period. A constant marginal utility is allotted to each of these indices; and each is given a present discounted value to allow for the date at which it will operate.

"Commercial" economic activities (i.e. those in which the goods and services produced are marketed) are described by vectors of fixed technical coefficients representing the unit consumption of intermediate products and skilled work at different levels, and also the gross investment required to increase production capacities.

The same applies to education considered as an activity; but the latter also includes a series of special inputs (and, hence, of special technical coefficients) namely students during their educational careers who are treated as "work in process".

The whole range of "commercial" activities is sub-divided into several sectors, and educational activity is divided into several educational levels (or "cycles"). The technical coefficients are arrayed in matrices: the columns are sectors or educational cycles, and the rows are different types of input.

Each activity, whether "commercial" or educational, is linked to a vector of activity variables which, in the case of the "commercial" activities, represents their total respective outputs and, in the case of educational levels, represents the number of pupils enrolled at each of these levels.

Although for present purposes education embraces all educational activities, both private and public, it is treated as "non-commercial", as the vast majority (eighty per cent) of French pupils and students attend State schools, and as private schools are State-subsidised.

Of all the non-commercial activities, that is, of all the public services and other non-profit-making institutions, education is the only one whose factors and products are treated as endogenous variables in this model. All other non-commercial activities are represented by exogenous variables.

Lastly, the programme is sequential in the sense that it covers several successive periods, its endogenous variables are dated for each of these periods, and some of them (production capacities, labour force and school population) are linked through time. There is, however, only one social preference function to be maximised which covers all the periods combined. Hence, there is a single programme in which the number of elementary constraints is multiplied by the number of periods taken into account.

2. Characteristics peculiar to the educational sector and to the training of skilled labour

In the model under review, education is regarded mainly as an industry producing the knowledge required by future workers of various skills.

The activity of the educational sector operates in at least four directions:

(a) It provides pupils with the knowledge essential for the general or occupational skills they will later possess as members of the labour force (including teachers and research scientists);

(b) It raises their cultural level and so influences the choices they will make and their ability to absorb fresh knowledge during their working lives;

(c) It develops scientific research within the universities themselves;

(d) Lastly, it helps to disseminate cultural, scientific and technical knowledge within the population as a whole through books and reviews, broadcasts, and the extra-mural activities teachers.

The education sector thus has an output of four related products which it would be interesting to measure and analyse accurately; but it is difficult to distinguish products (b), (c) and (d), and to find numerical indices for them. For the time being, therefore, the only account taken of them is to introduce a pre-assigned minimal growth constraint on the education services in order to preserve the general cultural and scientific needs of the population and the country, it being assumed that some young people who have been educated (some girls for instance) will not join the labour force. By contrast, the model is focussed on the output , and use by other sectors of activity, of the first product of the educational sector, that is, the knowledge needed for the training of skilled workers.

The output of the educational sector is thus regarded as consisting entirely of intermediate products. The product of the educational sector is considered only as input for the training of skilled labour, and no part of it goes into final personal consumption. These intermediate products are themselves broken down into knowledge levels corresponding to educational levels.

Choosing a numerical index which might validly represent the "products" of educational activity was not easy. It was ultimately felt that since the only product of education which the model took into account was the knowledge needed for the training of skilled workers, the most suitable numerical index was the number of pupils enrolled at a particular educational level at a given date.

Part of this product can be treated as a "final product",that is, the number of pupils who, at the end of a school year, leave the educational system to join the labour force (or remain unemployed). The other part, that is the number of pupils who continue their studies, consists of "work in process".

Pupils who have left the educational sector are treated as inputs of a notional activity - the training of skilled workers. They are not, however, the only input since the model adopts the principle that, in any period, the manpower stock at a given level of skill is fed from three sources: workers still in the labour force who had that level of skill before and have kept it, those still in the labour force who have risen to this level of skill by occupational proficiency and, lastly, young people drawn from the educational sector.

The "stock" of skilled workers built up from these sources constitutes the output of the notional sector already mentioned, and will serve as manpower resources for the endogenous "commercial" sectors and for the educational sector.

This procedure seemed to give a clearer and truer picture of the real state of affairs than would have been obtained had the levels of skill been by-passed, by classifying the labour force according to required educational level. At the level of aggregation adopted for this model, one-to-one correspondence could not be assumed, especially since occupational proficiency was taken into account.

The benefit of this more realistic approach is that the optimization model - the logic of which consists in minimising costs - will give first pick to those types of skilled labour training which are least costly.

There was a risk that the model might go too far in this direction and, in the absence of an adequate numerical index, educational "end-use" consumption (on cultural grounds, for instance) would not appear in the preference function. It was therefore necessary to give the educational sector an additional constraint whereby the percentage of young people continuing their studies can not fall below a certain minimum. If effective, this constraint reduces the social optimum, measured in terms of "commercial" consumption and of investment at present discounted values, but it implies an educational "surplus". The difference, depending on whether the social optimum is obtained with or without this constraint, is the price that the community pays - in terms of goods and services dispenses with - for this "surplus" education.

Finally, allowance is also made for another constraint peculiar to the educational system: one which makes the total expenditure of the system (either on an overall basis or for each educational level) subject to a budgetary ceiling. Again it will be interesting to compare the results when the social optimum is obtained with, and without, this constraint.

3. Treatment of technical progress and choice of techniques

A model for the optimum allocation of the physical and human resources of an economy, which lays stress on manpower skills and on education, might be expected to make rational choices not only between sectors, but also between technical processes within each sector. For the time being, at least, this is not so for our programme.

In this programme, technical progress makes an exogenous impact by modifying, over time, the coefficients of the input-output tables for intermediate consumption, manpower and fixed assets. However, these modifications are exogenous, and the coefficients in question are fixed for each period. The model as it stands at present does not accordingly allow any choice between techniques; and the costs of educating the labour force have no impact on the techniques adopted. These costs have a bearing only on the allocation of resources among endogenous sectors, and hence fix their outputs only in relation to the expense, in terms of required education, of the planned techniques.

The structure of the programme, however, enables it to incorporate interchangeable technical processes fairly easily. It could, in fact, make use of the conventional method which consists in introducing, within each sector, notional subsectors corresponding to the processes in question, each sub-sector being given an input-output coefficient vector representing the technique involved. In this case, the choice of one technique rather than another would indeed be swayed by the relative scarcities of the various factors involved, and more particularly by the relative scarcity of the existing resources of skilled manpower. Because the programme stretches over several periods, however, the educational sector (by training young skilled workers) relaxes this constraint all the more as the horizon of the model is more remote. Needless to say, the extent to which this constraint is relaxed in the long run itself depends on the resources allocated to education and, hence, ultimately on the comparative marginal yields of education and other activities, measured in terms of social satisfaction. This is the case in the present model.

4. Physical equilibrium and optimum prices

All the activity variables and constraints of the "primal" programme have a physical significance, even though the variables relating to goods are expressed in terms of constant monetary units for purposes of aggregation. The budgetary constraint on the educational sector is the only one of a financial nature. It follows that no parameter representing marginal costs or marginal yields is directly shown in the constraints of the "primal" programme.

With the dual programme, on the other hand, it is possible to calculate many prices and "rents" which, for our purposes, are called "notional prices"; and this lays the way for measuring the relative yields of educational investments.

II. VARIABLES AND CONSTRAINTS OF THE PRIMAL PROGRAMME

The detailed structure of the "primal" programme is explained below by reference to its general shape, that is, without specifying the dimensions which follow from the number of activities and periods taken into account.

A. The variables of the primal programme

1. Exogenous variables

The model treats some quantities as exogenous variables which may be classified in four categories:

1. The first category embraces all the variables relating to the initial period.
2. The second covers variables representing the activities of certain sectors, i.e. agriculture, government services and foreign trade, which cannot easily be brought into an optimization exercise of this type for the following reasons :
   Agriculture : In view of the technical characteristics of this sector and the protection and subsidies it receives, its output is little related to demand, and its manpower inputs depend more on the migration of manpower to other sectors and to the towns than on technical progress.
   Government services : For present purposes, the activity of these services is deemed to be determined by policy decisions. Some government services (Health, Transport, etc.) could be analysed on the same lines as education in the present case. It was precisely because our methods had first to be tested on

a simpler case (education) that this was not attempted forthwith.

Foreign trade Here, we must confine ourselves to the formulation of assumptions which, incidentally, could be tested with several parameters.

3. The third category of exogenous variables consists of demographic variables: pupils enrolled in the educational system; net immigration (by level of skill); rural migration to the towns.
4. The fourth category consists of the budgetary ceilings which may possibly be embodied in a programme, either for education expenditure taken as a whole, or for each educational level separately.

2. Endogenous variables

1. In principle, all the variables of the "primal" programme are physical quantities; but the aggregation of "commercial' goods and services makes it essential in most cases to treat these variables not as quantities in the strict sense but in terms of volume, that is, values at constant prices for a base year.

2. All the variables are dated by reference to a base period: a single year in the expanded model and a 3-year period corresponding to school "cycles" in the test model. A good many variables are flows per period (e.g. volume of 'commercial' goods and services, number of young workers beginning their occupational careers), the others being end-period "stocks" (manpower stock, school population stock). As a general rule, flows are shown as X and stocks as S.

3. The activity variables of the model represent the outputs and inputs of the various sectors taken into account. These variables fall into three major categories:

(a) "Commercial" goods and services; that is, so-called "products" in the French national accounting system. The symbol for these considered individually is i and, taken together, they constitue a vector which is numbered 1.

(b) Skilled workers and the quantities of work they produce. These variables are denoted by q (index of skill) and the vector in their case is numbered 2.

(c) The student population whose numbers relate to the educational level denoted by h. The entire school population is grouped in a vector which is numbered 3.

4. As already stated, "commercial" goods and services are measured in terms of volume, while skilled workers are measured in numbers. On the other hand, numerical indices for educational services are more difficult to choose.

As the "product" of education is restricted in this model to the knowledge required for the training of skilled workers, this output can be represented by the number of graduates or 'quasi-graduates' (i.e. non-graduates having an equivalent standard of knowledge) who leave the educational system each year and are added to the pre-existing stock of manpower. The number of students leaving the educational "activity" can thus be taken as a valid numerical index of the finished product which education delivers to the notional activity of skilled manpower training. This convention can reasonably be applied to all the services produced by the educational activity. The entire enrolled population by educational level during the period under review may therefore be taken as an index of the total quantity of these educational services.

The activity variables of the educational sector, thus expressed in numbers of pupils of different educational levels, apply only to those pupils about whom choices must be made; i.e., choices between enrolment and non-enrolment, or choices between several streams within the educational system. This means that pupils in the elementary "cycle" (aged 6 to 11 or so) do not appear in the model.

B The constraints of the primal programme

The primal programme in its matrix form has nine constraints for each of the periods considered. The first seven are essential for the solution of the programme and must be observed in any event. These are physical constraints ensuring that the available resources in "commercial" goods and services, skilled workers and educational services are greater than the sum total of their respective uses. The last two constraints are included for special purposes only. The eighth constraint represents a minimum growth requirement for educational services (further reference is made to its significance later on), while the ninth represents a budgetary ceiling which reflects the financial limitations imposed on education expenditure.

For each of the periods covered by the model, these nine constraints operate in such a way that if the model comprises T periods and nine constraints, this will give, in matrix form, 9xT constraints when the primal programme is worked out (1).

1. Constraints relating to resources and uses of commercial goods and services produced and of available labour

The first two constraints are of classical type: they conform to the rule whereby resources of goods and services cannot be exceeded by consumption, including consumption of education considered as an activity. The goods and services involved are "commercial" goods and services (constraint 1) and labour (constraint 2); $\left[A_1\right]$ and $\left[N_1\right]$ respectively represent the matrix of technical coefficients for current inputs and the matrix of labour input coefficients in the productive economy; $\left[A_3\right]$ and $\left[N_3\right]$ are matrices of similar types for the educational system.

As previously stated, the model determines only the optimum increase in personal consumption of "commercial" goods and services.

Initial personal consumption, (C_1 (0)), and part of the final demand for these same goods during the period considered, (D_1 (t)), are determined on an exogenous basis.

D_1(t) is a vector which gives, by product, the total sum of the consumption of public services other than Education of Financial Instructions and of the Agricultural Sector, of changes in stocks and of the balance of foreign trade.

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(1) A table giving the symbolic dimensions of the system is shown in Appendix I.

The elements ω of the diagonal matrix $\left[I - \hat{\omega}_2 \right]$ (constraint 2) represent the maximum rates of frictional unemployment which are regarded as acceptable.

2. Constraints relating to the inability to exceed production capacities

Constraints (3) and (4) express the fact that the output of commercial goods and services and of educational services is restricted by the production capacity available in the sectors relating to activities 1 and 3 during the period considered. This capacity is estimated in terms of units of the particular commodity to be produced.

The increase in capacity during the periods considered is determined by the model as follows: (The working of the system is explained with reference to activity 1, but entirely similar reasoning can be applied to activity 3).

Let I_{ji} (t-1) be the gross investment in capital asset j made in year for production of commodity i; and K_i (t - 1) be the increase in production capacity for commodity i resulting from investment I_{ji} (t - 1); this increase in capacity will be available only in the following period, t. The capital coefficient, b_{ji} an (element of the matrix B_1) is defined by the following relationship:

$$bji = \frac{I_{ji}\,(t-1)}{K_i\,(t-1)}$$

This coefficient differs from the classical (gross) marginal capital coefficient in that its denominator does not represent effective additional output, X_i (t) but increase in capacity. It would be wrong, however, to link the two coefficients in question by a third one representing the degree of utilisation of production capacity. In the model, this degree is not fixed, since the factors cannot be fully employed; and this is reflected by the form given to the constraints, i.e., "non-strict inequality".

In order that the model may be written more simply the variable

I_{ji} is not included, and gross investment is shown by $b_{ji} \dot{K}_i (t-1)$.

The requirements for commodity i are determined by the maximization under constraint of the preference function and so the increase in capacity needed for the production of this type of goods is also determined. This capacity is written off by the conventional straight line depreciation method.

(r_1 = rate of depreciation of capacity for activity 1)

(r_3 = rate of depreciation of capacity for activity 3).

The capacity available for each activity at the beginning of the first period is determined on an exogenous basis (the same applies to the values of all the variables at the beginning of this period).

Linkage between skilled manpower requirements and training

Constraints 5, 6 and 7 express the linkages between skilled manpower requirements, the training of this manpower and the activity of the educational sector.

3.1 A distinction is made between the manpower stock at the end of a period, ($S_2(t)$), and the flow of young people leaving the educational system and either entering the labour force, or not doing so, but in either case having acquired a certain level of skill, (X_2 (t)).

Activity 2 is not in the strict sense productive; it corresponds to the classification of individuals leaving the educational system and the promotion of those who are already part of the labour force. This classification comprises several stages which the model defines by the following operators:

$$(E_2), \quad (G_2) \text{ and } (\hat{\theta}_3)$$

(E_2), in constraint 6, is a matrix of transition between educational levels (h) and skill levels (q). It indicates the proportion of young people of different educational levels which seems likely to be required in future by the various levels of skill in the labour force.

It is thus partly normative but is also influenced by past results, as shown by the statistics for the skills and educational levels of the labour force.

(G_2) represents the shifts from one level of skill to another within the available manpower stock taken as a whole; these shifts follow from occupational upgradings and professional experience acquired in actual practice.

Constraint 5 therefore expresses the fact that the available labour force at time t, $S(t)$, cannot exceed the resource which is made up of the sum total of the following flows : $(G_2) \times (I-\delta_2) \times S_2(t-1)$, $X_2(t)$ and $Z_2(t)$.

$Z_2(t)$ represents the foreseeable additional flow of manpower at each level of skill for each period. This exogenous variable comprises young people leaving the educational sector before entering the French "Classe de 3ème" (the 4th form in the secondary cycle), ex - farmers transferring to endogenous "commercial" sectors and the ceiling figure for immigrant workers.

$(\hat{\theta}_3)$ is a diagonal matrix which, in constraint 6, is intended to distinguish the young people of a given educational level who are likely to join the labour force from those who, on leaving school, remain at home (this particularly applies to girls); in other words it represents the propensity of young people to join the labour force, and is measured by the rates of activity, by age and sex of this category of the population.

3.2 A particular difficulty concerning the educational sector is that it turns out not only finished products in the form of young graduates or "quasi-graduates" ($\left[E_2\right] X_2(t)$), but also, in the course of the same period, "work in process" in the form of students who will continue their studies during the subsequent period. It is consequently necessary to regard the entire school population in a given period, i.e. pupils who will leave the educational sector in the following year and those who will remain in it, ($S_3(t)$), as an activity variable for the educational sector.

3.3 Constraints 6 and 7 describe the internal flows in the educational system. Constraint 7 expresses the fact that any element, $s_h(t)$ of vector $S_3(t)$ is a utilization at level h of three available resources:

(a) The flow of "repeaters" remaining at the same level h at time t,

i.e. : $\rho_h(1-\delta_h) \quad s_h(t-1)$

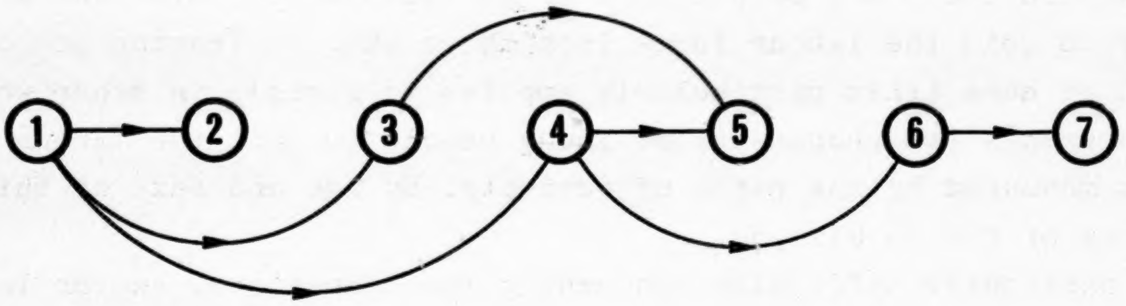
ρ_h, element of the diagonal matrix $\left[\hat{\rho}_3\right]$, is the "repeater" coefficient for level h.

δ_h, an element of the diagonal matrix $\left[\hat{\delta}_3\right]$, is the coefficient for attrition (death, etc.) of pupils of level h.

(b) Exogenous flow of pupils joining the educational system for the first time, i.e. $j_h(t)$, an element of vector $J_3(t)$.

(c) The flow of pupils joining level h and coming from levels h', linked to h, and represented by $-\left[P_3\right] \cdot Y_3(t)$.

Internal flows are represented by the following network :



The nodes represent levels h, and the arrows represent possible transitions between levels from one period to another. For instance, arrow (i,j) represents the possible transition of a pupil from level (i), at period (t - 1), to level (j), at period t.

Vector $Y_3(t)$ is made up of the flows passing along each arrow, as follows:

$$Y_3(t) = Y_{12}(t),\ Y_{13}(t),\ Y_{14}(t),\ Y_{35}(t),\ Y_{45}(t),\ Y_{46}(t),\ Y_{67}(t)$$

This graph is combined with its "characteristic matrix" or "incidence matrix", comprising as many rows as there are levels and as many columns as there are arrows or flows, $Y_{ij}(t)$, and having as elements:

$$a_{k,(ij)} = \begin{cases} +1, \text{ if } i = k \\ \\ -1, \text{ if } j = k \\ 0, \text{ otherwise} \end{cases}$$

The incidence matrix of the foregoing graph will therefore be:

Levels Arcs

$$A = \begin{array}{cccccccc} & (12) & (13) & (14) & (35) & (45) & (46) & (67) \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & -1 & 0 & 1 & 1 & 0 \\ 5 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 7 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{array}$$

This matrix A is broken down into two matrices, as follows:

$$A = \left[M_3\right] + \left[P_3\right]$$

where $\left[M_3\right]$ = Positive incidence matrix based on A, taking into account only coefficients equal to 1;

$\left[P_3\right]$ = Negative incidence matrix based on A, taking into account only coefficients equal to -1.

Hence $\left[P_3\right] Y_3(t)$ represents the flows entering each level h, and will appear in constraint 7;

$\begin{bmatrix} M_3 \end{bmatrix} Y_3$ (t) represents the flows emerging from each level h, and consequently appears in constraint 6 which describes the various following utilizations of the flow of "non-repeaters" $(1-\rho_h)$

$(1 - \delta_h)\ s_h\ (t - 1)$ at level h =

(a) An endogenous flow of pupils, $e_{hq}\ x_q(t)$, having received education of level h during period t, and leaving to join the non-agricultural productive economy;

(b) A flow determined on an exogenous basis, $d_h(t)$, which expresses the demand for pupils of level h to meet the requirements of government services and agriculture;

(c) The flow of pupils leaving level h for higher levels, h", linked to h, and represented by $\begin{bmatrix} M_3 \end{bmatrix}\ Y_3(t)$.

In contrast to the procedure adopted for certain models (1), the programme is not made subject to any predetermined coefficient for progression from one class or cycle to another, nor to any coefficient for school-leavers joining the labour force. The difference in the case of our model is that, by optimization, it remains free to determine for itself the breakdown of the "non-repeating" school population between school-leavers joining the labour force and progression from one level to another within the educational system.

In this connection, it was felt that this breakdown depended at least as much on economic factors as on pedagogic considerations, and that it would accordingly be useful if the model made this clear. On the other hand, the rates for "repeating", $\begin{bmatrix} \hat{\rho}_3 \end{bmatrix}$, and attrition, $\begin{bmatrix} \hat{\delta}_3 \end{bmatrix}$, (and hence, implicitly, the rates of progression from class to class) are imposed exogenously on the model, since they are treated depending mainly on pedagogic and demographic factors.

4. Possible additional constraints

The possible additional constraints are numbered 8 and 9. Constraint number 8 introduces a minimum growth rate for education, while constraint number 9 introduces a ceiling for budget expenditure.

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(1) For instance the well known "Planning Model for the Educational Requirements of Educational Development" by J. Tinbergen and H.C. Bos (OECD "Econometric Models of Education", 1965).

(a) Minimum growth rate for education (8)

This constraint answers the need for the continued development of education on cultural, social and civic grounds, and also for economic purposes. It does indeed appear that the prospect of technical progress and the resulting reconversion of activity call for more elaborate general education to facilitate such reconversion.

This new constraint, like those already mentioned, is physical. It strengthens constraint number 6 by imposing, on variable $S_3(t-1)$, a diagonal matrix for the minimum rates of progression of pupils from one educational level to another, $\left[\hat{\pi}_3\right]$, which compels "non-repeaters" to remain in the educational sector.

The policy aspect of this constraint and of the minimum rates of "retention" in the educational sector is obvious. It would be useful to work out parameters for the value of $\left[\hat{\pi}_3\right]$ in order to study the effects of the various policies which might be followed.

(b) Budgetary ceiling for education expenditure (9)

This financial constraint brings in a single matrix, $\left[\hat{f}_3\right]$, a matrix of the real costs of the various educational levels. Its second component, F_3 (t), represents the total aggregated budget specified by the Minister of Finance or by the Government when appropriations are made for the budget of the Ministry of Education or for the "Schools Equipment Plan". The vector form given to F_3 (t), to ensure formal consistency with the rest of the model, implies that the aggregated budget must be broken down by level of education, h. If the envelope applies to total educational expenditure only, F_3 (t) then simply becomes a scalar.

In this case too, the political aspect of the constraint is obvious; the testing of various parameters is accordingly called for, and they may even imply elimination of the constraint.

The impact of the additional constraints may be either effective, or superfluous (if their purpose is already fulfilled by the interplay of the first seven constraints). In the former case, the optimum attained - that is, the màximum value of the social utility function - is lower than it would be if only the seven first constraints had been brought into play. In this way, it is possible to show, in quantitative terms, the effects of the application of different polic e., one making the development of educational expenditure a priority issue (constraint number 8), the other, by contrast, aiming to limit such expenditure (constraint number 9).

C. The output functions of activity sectors

The output functions of the various sectors can be seen by reading vertically the elements included in the constraints of the "primal" model. These are illustrated by the graph which appears in Appendix III.

1. Output function for "commercial" goods and services

This output function brings in the matrices of technical coefficients of columns 2 and 3 relating to variables $X_1(t)$ and $K_1(t)$. The impact of intermediary products is shown by $\left[A_1\right]$, and that of labour by $\left[N_1\right]$.

The impact of fixed assets follows from the effect of $\left[B_1\right]$ combined with successive investments made at various times after the initial period.

2. Output function for skilled manpower

This output function appears in columns 4 and 5. Column 4 makes it possible to move from the skilled manpower stock inherited from the previous period, $S_2(t-1)$, to the maximum available quantity of skilled manpower, $\left[I - \hat{\omega}2\right] S_2(t)$. Column 5 shows that the knowledge (classified by educational level) possessed by young people leaving the educational system can be converted into skills available for employment in the commercial productive economy of the non-agricultural sector, ($\left[E_2\right]$, $\left[\theta_3\right]$).

3. Output function for the educational sector

The output function specific to activity 3 is shown in column 6. The "production process" here comprises several offshoots turning out intermediary products which do not appear at the final stage (columns 7 and 8).

This output function describes the use of a number of inputs for obtaining two outputs.

(a) Inputs at time t

1. Supplies and materials consumed during period t:
   $\left[A_3\right] S_3(t)$ (constraint number 1).
2. Skilled labour supplied by teaching staff:
   $\left[N_3\right] S_3\ (t)$ (constraint number 2).
3. Capacity of educational equipment, measured in units of the commodity to be produced: $\left[I\right] S_3(t)$ (constraint number 4).
4. Flow of pupils to whom teaching is dispensed:
   $\left[I\right] S_3(t)$ (constraint number 7).

(b) Outputs at time t + 1

Output becomes available at time t + 1 because a period of time was needed to produce it. The productive operation carried out consists in dividing the input at time t, $S_3\ (t)$, into two parts which appear in constraints 6 and 7 of period t + 1 of the primal programme.

1. "Repeaters" in period t + 1: $\left[\rho_3\right]\left[1 - \hat{\delta}_3\right] S_3(t)$
2. "Non-repeaters" in period t + 1 : $\left[1 - \hat{\rho}_3\right]\left[1 - \hat{\delta}_3\right] S_3(t)$
   These "non-repeaters" may then either continue their studies or join the labour force.

D. Comments on the last period

The last period calls for special treatment as regards the output of capital equipment and the output of the educational sector, these two items having similar features. The end period or "horizon" is shown as T.

The amount of investment in commercial goods and services during the final period, ($\left[B_3\right]\dot{K}_3(T)$), and ($\left[B_1\right] K_1(T)$), cannot be determined by the model, as its effects will not be felt until after the horizon. This is therefore an exogenous datum factor ($I_1(T)$, constraint number 1). Similarly, individuals joining the educational system during the end period, but not leaving it before the horizon is reached, represent a potential output of skilled labour. However, this potential will not begin to operate until after the horizon, and the model cannot therefore assess the increased satisfaction, in terms of goods and services that will follow from it. The value of this potential in the final year must consequently be determined exogenously. If not determined in this way, the value of $S_3(T)$ could quite well be zero, which is not desirable. It will, however, be necessary to check whether the exogenous value of $S_3(T)$ is not unrealistic, having regard to the values given by the model to the population in previous years: $S_3(T-1)$, etc.

III. THE SOCIAL PREFERENCE FUNCTION OF THE PRIMAL PROGRAMME AND THE GENERAL FEATURES OF THE DUAL PROGRAMME

A. The social preference function of the primal programme

The structure of this model is based on its objective, which is to maximize the present discounted value - over all the periods considered - of the sum total of the variables representing the standard of living of the population of the production potential of goods and services, of skilled labour and of education continuing beyond the time limit (horizon) of the model, and so enabling the process of growth to continue beyond that limit. In this way, provision is made for meeting the consumption requirements of the population over all periods covered by the Plan and for safeguarding the production potential and the skilled labour potential beyond that point. However, the numerical indicators that will represent the different elements, and the choice of marginal utilities with which they will be weighted in the preference function must be specified.

1. Standard of living indicators

As a general rule, the standard of living indicators are quantitative indicators of personal consumption of commercial goods and services, educational services and other collective services. However, this rule had to be mitigated in order to overcome a number of difficulties.

1.1 If the vector of personal consumption (i.e. consumption broken down by category of products) had been included as variables to be maximised in the target function and in the constraints, at least two risks would have arisen. In the first place, the model might reduce the volume of total personal consumption - which would be absurd. Secondly, the model might distort the product structure of this consumption, and this is no more tolerable. To eliminate the first of these hazards, increases of personal consumption through time are alone taken into account. These increases are made subject to the condition that endogenous variables are non-negative to avoid the risk of a decline in personal consumption. For the time being, the model does not introduce any bottom limit for growth of consumption. It was considered useful to let it freely determine the personal consumption compatible with its own optimum. A bottom limit of this kind could be reintroduced later, if required. For the time being, the preference function aims only to maximise the present discounted value of the sum total of the successive increases in personal consumption, starting from its initial level. It is therefore the increase in consumption variable which appears as an endogenous component both in the preference function and in the first constraint. Furthermore, to prevent any abnormal distortions in the pattern of consumption, the trend of this pattern over time is assumed, the growth of consumption being, of course, taken for granted. However, the essential feature from a methodological standpoint is that this structure is treated as exogenous.

1.2 The educational services and final collective services to be absorbed by personal consumption are dealt with as follows: in the case of educational services, a minimum growth rate for education is introduced as a possible additional constraint (constraint N° 8), while final collective services are treated as exogenous variables. Only in a far more extensive model, embracing all collective requirements and services, would it be possible to try to make the other collective services subject to a treatment similar to that applied to education in this case.

2. Production potentials of sector 1 and 3

2.1 The production potential of goods and services which must survive at the end of the period to cater for future needs is represented by the present discounted value of the services that the investments, made over the whole time-range of the model, will yield beyond that limit.

Part of the investments in question are lost in an intermediary way in the course of the periods considered, and must not therefore appear in the target function. On the other hand, the residual part will be consumed only after the end of the time-range (horizon), and hence remains an element of final demand for commercial goods and services over all the periods covered by the model. Failure to optimize this residual part would cause an under-estimation of the returns on investment, and so distort the choice between consumption and investment which the model simulates. Allowance is made for this factor by introducing, in the preference function, the residual value, discounted to present value, of the capital equipment put into the productive sector throughout the time-range covered by the model.

2.2 The same problem arises with regard to the residual production potential relevant to education; and it is dealt with in the same way.

3. Skilled labour potential

Similar reasoning applies to individuals leaving the educational system to join the labour force during the successive periods, $\sum_t E_2 \; X_2(t)$. Their working lives normally extend beyond the time-limit adopted for the model. Here again, if the services they will render in the productive sector, or in the educational sector after that time, were not taken into account, the "training effort" imposed on the educational sector would be undervalued.

The preference function must therefore include the present discounted value of the services that will be rendered, beyond the time-range covered by the model, by the outflows of individuals from the educational system to the labour force during those periods covered by the programme.

4. The marginal utilities used as weights for the variables of the preference function

The weights for the variables of the preference function of the primal programme represent the marginal social "desirability" of each variable, it being assumed for our purposes that this "desirability" is assessed by the community. In the case of commercial goods and services, if it is accepted that the operation of markets in actual practice is not too remote from the conditions of perfect competition, it is possible - as a first approximation - to take market prices as valid numerical indicators of these marginal "desirabilities". This is in fact what implicitly happens with this model, insofar as the aggregations in terms of volume bring in the market prices of a base period. However, these prices do not explicitly appear in the utility function, nor, consequently, in the constraints of the dual programme. As already explained our argument is based on total personal consumption allocated over time. A discount rate enables us to add together these different total consumptions which refer to different time periods. It is this discount rate which is used as a coefficient for weighting additional personal consumption in our social preference function.

The coefficients $\mu(t)$ represent the residual values of the physical or human assets built up as a consequence of the investments made during the periods covered by the model.

The residual value, $\mu_1(t)$, of the increases in capacity, $\dot{K}_1(t)$, achieved during the periods considered, in the sectors producing commercial goods and services, is (by definition) equal to the discounted sum total of the future services rendered by these increases beyond the time range of the model.

The calculation of $\mu_1(t)$ is based on a principle of economic returns: for the purposes of the optimum adopted by the model, the sum total of unit "rents" (i.e. returns), $V_1(t)$, paid during the periods, and the unit recovery value, $\mu_1(t)$, to be paid for the services rendered per unit of increased capacity, must be equal to the cost of the investment, $[B_1]'U_1(t)$, which brings about the said increase (see dual constraint N° 3 - Appendix III). In fact, in the model under review, the discounted cash flow of an investment operation leaves a balance of zero.

It is then assumed that the dual relative prices, $U_1(t)$, remain stable, and that capacities are written off at constant yearly rates:

$$V_1(t) = \frac{\left[B_1\right]' U_1(t)}{n}$$ when not considering the depreciation rate, r_1, nor the discount rate

(where n is the working life of the asset).

$\mu_3(t)$, the residual value of school and university equipment has the same significance for educational activity, and is calculated in the same way.

The calculation of $\mu_2(t)$ (the residual value of the skilled manpower assets trained during previous periods) is based on the interpretation of dual constraints of type N° 4 and type N° 5. The additional workers, $X_2(t)$ trained by the educational sector during the periods under review are regarded as capital equipment resulting from the investments made in the educational system (in terms of materials, staff and fixed assets) by the economy.

This capital equipment is written off at constant yearly rates up to the horizon of the model. $U_2(t)$ is depreciation for each period for the purposes of evaluation it is taken as equivalent to the wages paid to a person, whose level of skill is q, for period t. The cost of this equipment is $V_2(t) + \mu_2(t)$ The part which remains to be written off after the horizon of the model is $\mu_2(t)$.

From the foregoing definitions (based on the interpretation of the dual programme) it follows that $\mu_2(t)$ is the sum total - discounted by reference to the horizon of the model - of the wages paid from that horizon until the end of the individual's working life, allowance also being made for the probabilities of occupational promotion, $\left[G_2\right]$, or elimination, $\left[\delta_2\right]$. Wage stability is assumed throughout all the periods considered.

It is clear that, in one way or another, all the marginal utilities of the different variables included in the social preference function depend on the discount rate. This rate of discount is determined on an exogenous basis; its constant relationship to the values of the variables in the function confirms the linear character of the latter.

B. The dual system: a few comments

The value of the preference function of the dual system is at its optimum when minimized. In the model under review, this value represents the minimal cost to the national community of the expenditure required to operate the system. Such expenditure corresponds to the allocation of productive forces and factors among the sector resulting from the optimization of the primal programme.

The dual variables can be interpreted in terms of prices. These are not observed prices but theoretical ones, so-called "notional" or "optimal" prices, corresponding to theoretical conditions of perfect competition or perfect planning. All the dual variables together with their precise definitions are listed in Appendix II.

The dual constraints of types N° 1, N° 2 and N° 3 concern commercial goods and services and have no unusual features. The position is different where skilled labour and educational services are concerned.

The dual constraint of type N° 4 expresses the depreciation at constant yearly rates, for periods t to T, of the investment cost for Activities 1 and 3 of the "capital equipment" (or "human asset") constituted by an individual at skill level q. These yearly instalments represent the "reference wages" of the individual concerned for each period.

The constraint of type N° 5 links the cost of educating an individual of educational level h to the cost of educating the same individual, with a skill level q, for activities 1 and 3. The constraint of type N° 6 puts the components of the cost of education in an explicit form.

The constraint of type N° 7 is simply an adaptation to the educational system of the constraint whereby the output of a sector is limited by the capacities available in that sector (see constraint of type N° 3 for activity 1).

In the constraint of Type N° 8, the notional price of the individual leaving level h at the end of period (t-1) exceeds or matches the price which the higher level h' must pay to take in this individual at the beginning of period t.

CONCLUSIONS

The theory of the optimization model for education has been explained; the possibilities for its application must now be considered. These possibilities relate, first, to the numerical evaluation of the parameters for operating the model, and secondly, to the use of the model for decision-making.

1. Transition from the theoretical model to the applied model

If our theoretical model is to be re-written in practical terms, the statistical data for estimating the parameters used as matrical operators must first be compiled. The collecting and processing of these statistics is now in progress. It covers the data available from the National Statistics Institute, the Economic Forecasting Directorate of the Ministry of Finance, the Ministry of Education and the "Commissariat Général du Plan" (Board of Commissioners for the Development Plan). However, this data is not always suited to the structure of the model; its main deficiencies concern the more remote "horizons" that can be envisaged beyond the time range of the Fifth Plan. It is, in any event, difficult to estimate the long term trend of the structural parameters of the model, whether for the classical input-output coefficients, the manpower coefficients by level of skill or, even worse, for the alignment of levels of education with levels of skill.

Before trying to build up a model on an extensive scale, it was accordingly considered necessary to construct and work out a smaller programme. This programme, known as the "test model" is now being formulated. It will comprise two non-agricultural "commercial" sectors, five levels of skill and seven educational levels. It will not apply to twelve monthly periods, but will confine itself to three yearly periods. This being so, the test model will comprise 123 "primal" constraints.

This test model will enable us to see the working of a programme geared to the definitions and characteristics set out in this paper, and to compare the results obtained by formalization with the results of the discretionary projections made by the "Commissariat du Plan" and by INSEE (National Institute of Statistics and Economic Studies), providing that their "horizon" is close to ours. Above all, the formulation of this test model will enable us to see more clearly how sensitive a programme of this kind can be to changes in parameters and exogenous variables, and even to the introduction of possible constraints.

Depending on the results of the test model, various possible refinements will doubtless come to light. For instance, it would be possible to introduce certain structural changes in the general model, to expand its dimensions by taking account of more sectors in the "commercial" economy or even in other activities, or to take into account groups of techniques which would be related to levels of skill and education among which the model would endeavour to make an optimum choice.

2. Possible utilizations

Though we believe in the usefulness of formalization, and in the value of the conclusions that can be drawn from it, we are not dazzled by its merits. Neither this model, nor any other of its kind, can automatically work out the decisions to be made by the authorities in the field of educational economies. By its very nature, the model inevitably simplifies the issue: it cannot bring in every factor, particularly those social and psychological factors which have a bearing on the economics of education and on decision making in this field. Furthermore, its linear form makes its solutions somewhat blunt; the reactions of the model are sometimes rough-edged. There cannot therefore be any question of letting it shape the course of educational policy. Here, as elsewhere, the purpose of a model - and especially an optimization model - is above all to get a better insight into the interdependence of the subject analysed, and to set out more systematically and coherently their dynamic relationships. It also helps to clarify the decisions of economic policy-makers by analysing the consequences of the projects they have to formulate and by making it easier to study the sensitivity of the economic system to changes in exogenous factors.

The sensitivity of the model has to be investigated because of the simplifications introduced in its construction and the sometimes serious uncertainties affecting the form of the relations, the values of parameters, the change of these values over time, and the values of exogenous variables.

In due course, we therefore propose to make a series of calculations in order to trace the operators and exogenous variables which play a strategic part in the model so calling for particularly close attention. We also intend to make systematic parametric tests concerning these key operators and exogenous variables in particular, in order to measure the sensitivity of the model to the variations thus introduced.

With this model we will also attempt something else: a systematic analysis of the economic consequences of the various policies which might be adopted for the development of education. This means that we will have to work out variants for those exogenous variables which are in the nature of economic policy targets, and to go closely into the effects of the introduction of the possible constraints Nos. 8 and 9 discussed above.

It will also be possible, if need be, to substitute for the optimization procedure an exogenous treatment of the variables contained in the preference function (increase in personal consumption and increase in the residual value of production capacities), in order to change the purpose of the model from optimization to simulation. The problem would then consist of studying, with the help of the model, the consequences of pre-assigning specific targets for the growth of personal consumption and of the production potential of the national community. Here again, of course, variants could be introduced and investigated with reference to different levels for the targets already mentioned.

To come back to the optimization model, the solution of the dual programme should enable us to study the relationships between optimized prices and costs, and to translate, in terms of notional "quasi-commercial" units, the social utility and cost of education considered as an "activity".

It is accordingly hoped that a model of this kind may be of interest not only for economic policy, but also for theoretical analysis.

Appendix I

Dimensions of the primal programme

(Number of constraints)

	Type of constraint	Type of limiting resource	Index of limiting resource	Number of constraints
	1	Commercial goods and services	i	I
	2	Skilled workers	q	Q
	3	Commercial goods and services	i	I
	4	Individuals having attained educational level h	h	$H - 1$
	5	Skilled workers	q	Q
	6	Individuals having attained educational level h	h	H
	7	"	h	$H - 1$
per period not including optional constraints			$2i + 2q + 3h$	$2I + 2Q + 3H - 2$
	8	Individuals having attained level of training h	h	H
	9	Budgetary resources "ceiling"	h or 1	H or 1
per period			$2i + 2q + 5h$ or $2i + 2q + 4h + 1$	$2I + 2Q + 5H - 2$ or $2I + 2Q + 4H - 1$

Appendix II

KEY

The model is written in matrix form. The coefficients shown in the left hand components of the constraints are therefore matrices (some of them diagonal). The endogenous variables, both primal and dual, and the second components (exogenous variables), are vectors.

General code

1 Index of the activity producing commercial goods and services
(i: branch index)

2 Index of skilled manpower training activity
(q: index of level of skill)

3 Index of the activity producing educational services
(h: index of educational level; 1: index of transition flows between educational levels)

t Index of period considered
($t = 1,\ldots\ldots,T$)

τ Index of intermediary period, used for summation
($\tau = 1,\ldots\ldots,t$)

Coefficients

A_1	matrix of input-output coefficients for activity 1 (element a_{ij})
A_3	matrix of coefficients of consumption of commodity 1 by activity 3 (element a_{ih})
B_1	matrix of coefficients of capital specific to activity 1 (element b_{ji})
B_3	matrix of coefficients of capital specific to activity 3 (element b_{hi})
N_1	matrix of manpower coefficients in activity 1 (element n_{qi})
N_3	matrix of manpower coefficients in activity 3 (element n_{qh})
E_2	matrix of coefficients for converting a level of skill q to an educational level h (element e_{hq})
$\hat{\theta}_3$	diagonal matrix of activity rates, by age and sex, of individuals emerging from the educational system and fit for employment (element θ_{hh})
$\hat{\omega}_2$	diagonal matrix of minimal frictional unemployment rates (element ω_q)
$\hat{G}_2$	diagonal matrix of coefficients for upgradings from one skill category to another (element g_{qq})
$\hat{M}_3$	matrix of positive incidence between educational levels (element + 1 or 0)
P_3	matrix of negative incidence between educational levels (element - 1 or 0)
$\hat{\rho}_3$	diagonal matrix of "repeating" coefficients (element ρ_{hh})
$\hat{\pi}_3$	diagonal matrix of minimum transition rates between educational levels (element π_{hh})
$\hat{f}_3$	diagonal matrix of real costs per pupil enrolled per educational level (element f_{hh})
$\hat{r}_1$	diagonal matrix of rates of depreciation of capital equip-

	ment in sector 1
$\hat{r}_3$	diagonal matrix of rates of depreciation of capital equipment in sector 3
$\hat{\delta}_2$	diagonal matrix of the rates of attrition of the labour force in sector 2
$\hat{\delta}_3$	diagonal matrix of the rates of attrition of school population in sector 3
α	discount rate
$\beta_1(t)$	increases in personal consumption discounting function
$\mu_1(t)$	residual value at the end of the time range (horizon) of capital equipment specific to activity 1
$\mu_3(t)$	residual value at the end of the time range of capital equipment specific to activity 3
$\mu_2(t)$	residual value at the end of the time range of individuals joining the labour force at period t
$\Gamma_1(t)$	pattern of consumption at period t of commodities produced by activity 1

Second components (exogenous)

$C_1(0)$	initial personal consumption of commodity 1
$D_1(t)$	other elements of final demand to be met in t^{th} period
$K_1(0)$	existing production capacity in the sectors of activity 1 during the base period
$K_3(0)$	existing production capacity in the sectors of activity 3 during the base period
$Z_2(t)$	exogenous variation of the manpower stock during the t^{th} period
$D_3(3)$	demand for skilled labour from exogenous sectors during the t^{th} period
$J_3(t)$	flow of individuals joining the educational system in period t

F_3 (t)	budgetary appropriation for education, by educational level, for period t

Primal variables

$X_1(t)$	output in t^{th} period of commodities produced by activity1
$K_1(t)$	growth of capacity achieved in the sectors of activity 1 at period t
$K_3(t)$	growth of capacity achieved in the sectors of activity 3 at period t
$C_1(t)$	growth of personal consumption from period t - 1 to period t
$X_2(t)$	flow of individuals joining the labour force at period t
$S_2(t)$	labour force at period t
$S_3(t)$	individuals being trained in the educational system in period t
$Y_3(t)$	transitional flow of school population from one educational level to another

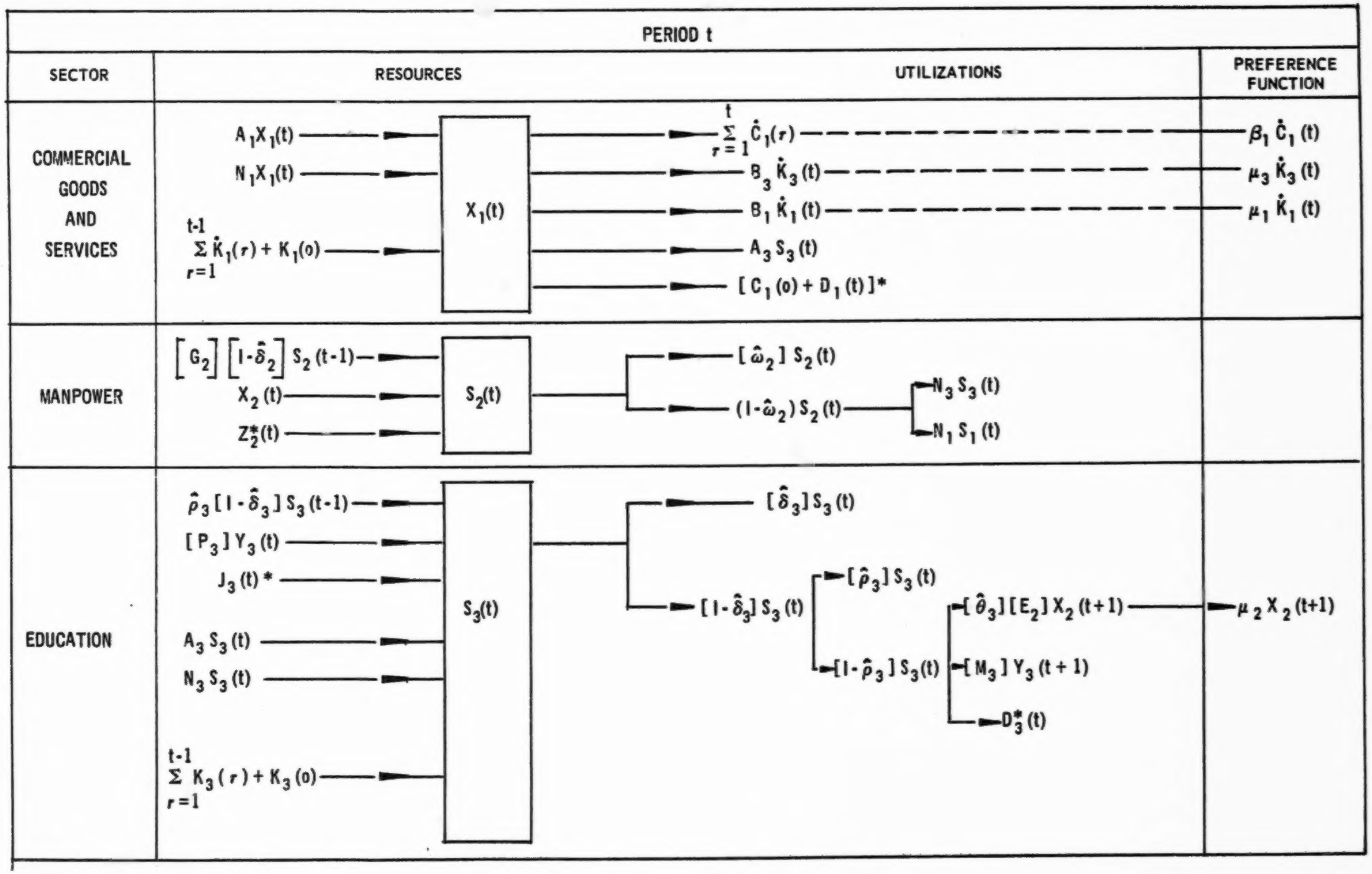
Dual variables

$U_1(t)$	base period price per unit, in time t, of goods and services produced by activity 1
$U_2(t)$	base period wages per unit, in time t, by level of skill
$V_1(t)$	base period "rent" per unit (provision for depreciation) paid, in time t, for the employment of capacity available in activity 1
$V_3(t)$	base period "rent" per unit paid, in time t, for the utilisation of capacities available in activity 3
$V_2(t)$	overall notional amount that activities 1 and 3 must theoretically allocate to reserves for the use of the

	human asset constituted by an individual of level of skill q during the period from t to T and the payment his wages. Hence, this is the sum total of the wages paid to this individual from t to T, and the write-off of his capital cost.
$U_3(t)$	cumulative cost of the education of an individual having reached educational level h in period t. It is also the shadow price at which the educational system sells the human asset which an individual of educational level h represents when he joins the labour force in period t + 1
$W_3(t)$	shadow selling price of an individual emerging from educational level h at the end of period t and "sold" to the next higher educational level h' at the beginning of period t + 1
$\Lambda_3(t)$	increment on the shadow price (U_3) at which human assets are sold by the educational sector to sector 2, this increment being due to the reduced supply of such assets which follows from the greater number of individuals kept within the educational system
$\xi_3(t)$	cost of the primal constraint which results from putting a "ceiling" on financial resources in the educational system. This is therefore, as it were, a "scarcity rent" on these financial resources.

Appendix III

RESOURCES AND UTILIZATIONS IN THE MODEL AT PERIOD t:


PERIOD t
SECTOR
RESOURCES
UTILIZATIONS
PREFERENCE FUNCTION
COMMERCIAL GOODS AND SERVICES
$A_1X_1(t)$
$N_1X_1(t)$
$\sum_{r=1}^{t-1} \dot{K}_1(r) + K_1(0)$
$X_1(t)$
$\sum_{r=1}^{t} \dot{C}_1(r)$
$B_3\dot{K}_3(t)$
$B_1\dot{K}_1(t)$
$A_3S_3(t)$
$[C_1(0) + D_1(t)]*$
$\beta_1\dot{C}_1(t)$
$\mu_3\dot{K}_3(t)$
$\mu_1\dot{K}_1(t)$
MANPOWER
$[G_2][1-\hat{\delta}_2]S_2(t-1)$
$X_2(t)$
$Z_2^*(t)$
$S_2(t)$
$[\hat{\omega}_2]S_2(t)$
$(1-\hat{\omega}_2)S_2(t)$
$N_3S_3(t)$
$N_1S_1(t)$
EDUCATION
$\hat{\rho}_3[1-\hat{\delta}_3]S_3(t-1)$
$[P_3]Y_3(t)$
$J_3(t)*$
$A_3S_3(t)$
$N_3S_3(t)$
$\sum_{r=1}^{t-1} K_3(r) + K_3(0)$
$S_3(t)$
$[\hat{\delta}_3]S_3(t)$
$[1-\hat{\delta}_3]S_3(t)$
$[\hat{\rho}_3]S_3(t)$
$[1-\hat{\rho}_3]S_3(t)$
$[\hat{\theta}_3][E_2]X_2(t+1)$
$[M_3]Y_3(t+1)$
$D_3^*(t)$
$\mu_2X_2(t+1)$

* Exogenous variables.

APPENDIX IV (PRIMAL) : Overall combination of constraints relating to period t, (t = 1, ... T)

			Dual Variables
i (1)	$\sum_{\tau=1}^{\tau=t}\Gamma_1(t)\dot{C}_1(\tau)-\left[I-A_1\right].X_1(t)+\left[B_1\right]\dot{K}_1(t)\quad+\left[A_3\right]S_3(t)\quad+\left[B_3\right]\dot{K}_3(t)$	$\le -\left(C_1(o)+D_1(t)\right)$	$U_1(t)$
q (2)	$\left[N_1\right].X_1(t)\quad-\left[I-\hat{\omega}_2\right]S_2(t)\quad+\left[N_3\right]S_3(t)$	≤ 0	$U_2(t)$
i (3)	$X_1(t)-\sum_{\tau=1}^{\tau=t-1}\left[I-(t-\tau-1)\hat{i}_1\right].\dot{K}_1(\tau)$	$\le \left[I-(t-1)\hat{i}_1\right].\dot{K}_1(o)$	$V_1(t)$
h (4)	$S_3(t)-\sum_{\tau=1}^{\tau=t-1}\left[I-(t-\tau-1)\hat{i}_3\right].\dot{K}_3(\tau)$	$\le \left[I-(t-1)\hat{i}_3\right].K_3(o)$	$\dot{V}_3(t)'$
q (5)	$S_2(t)-\left[G_2\right]\left[1-\hat{\delta}_2\right]S_2(t-1)-X_2(t)$	$\le {}_2(t)$	$V_2(t)$
h (6)	$\left[\hat{\theta}_3\right]\left[E_2\right]X_2(t)\quad-\left[I-\hat{\rho}_3\right]\left[I-\hat{\delta}_3\right]S_3(t-1)\quad+\left[M_3\right].Y_3(t)$	$\le -D_3(t)$	$U_3(t)$
h (7)	$S_3(t)-\left[\hat{\rho}_3\right]\left[I-\hat{\delta}_3\right]S_3(t-1)\quad+\left[P_3\right].Y_3(t)$	$\le J_3(t)$	$W_3(t)$
h (8)	$\left[\hat{r}_3\right]\left[I-\hat{\rho}_3\right]\left[I-\hat{\delta}_3\right]S_3(t-1)\quad-\left[M_3\right].Y_3(t)$	≤ 0	$\Lambda_3(t)$
(h) (9)	$\left[\hat{f}_3\right]S_3(t)$	$\le F_3(\ )$	$\xi_3(t)$

$$\text{Max }\Psi=\sum_{t=1}^{\tau=T}\beta_1(t).\dot{C}_1(t)\quad+\sum_{t=1}^{t=T-1}\mu_1(t).\dot{K}_1(t)\quad+\sum_{t=1}^{t=T}\mu_2(t).X_2(t)\quad+\sum_{t=1}^{t=T-1}\mu_3(t).\dot{K}_3(t)$$

APPENDIX V (DUAL) : Overall combination of periods

					Primal Variables
t	(1)	$\sum_{\tau=T}^{\tau=T}\Gamma_1'(\tau)U_1(\tau)$	$\geq$	$R_1(t)$	$\dot{C}_1(t)$
$t \times 1$	(2)	$-[I-A_1]'U_1(t) + [N_1]'U_2(t) + V_1(t)$	$\geq$	0	$X_1(t)$
$(t-1) \times 1$	(3)	$[B_1]'U_1(t) \qquad \sum_{\tau=t+1}^{\tau=T}[I-(\tau-t-1)\hat{\imath}_1]V_1(\tau)$	$\geq$	$\mu_1(t)$	$K_1(t)$
$t \times q$	(4)	$-[I-\hat{\omega}_2]'U_2(t) \qquad +V_2(t) - [I-\hat{\delta}_2]'[G_2]'V_2(t+1)$	$\geq$	0	$S_2(t)$
$t \times q$	(5)	$-V_2(t) \qquad + [E_2]'[\hat{\theta}_3]'U_3(t)$	$\geq$	$\mu_2(t)$	$X_2(t)$
$(t-1 \times h$	(6)	$[A_3]'U_1(t) + [N_3]'U_2(t) \qquad + V_3(t) - [I-\hat{\delta}_3]'[I-\hat{\rho}_3]'U_3(t+1) - [I-\hat{\delta}_3]'[\hat{\rho}_3]'W_3(t+1) + W_3(t) + [I-\hat{\delta}_3]'[I-\hat{\rho}_3]'[\hat{r}_3]'\Lambda_3(t+1)+[\hat{f}_3]\xi_3(t)$	$\geq$	0	$S_3(t)$
$(t-1) \times h$	(7)	$[B_3]'U_1(t) \qquad -\sum_{\tau=t+1}^{\tau=T}[I-(\tau-t-1)\hat{\imath}_3]V_3(t)$	$\geq$	$\mu_3(t)$	$\dot{K}_3(t)$
$t \times 1$	(8)	$[M_3]'U_3(t) \qquad + [P_3]'W_3(t) - [M_3]'\ \Lambda_3(t)$	$\geq$	0	$Y_3(t)$

$$\text{Min } \Phi = -\sum_{t=1}^{t=T-1}\Big(C_1(o) + D_1(t)\ 'U_1(t)\Big) + \sum_{t=1}^{t=T} K_1'(o)[I-(t-1)\hat{\imath}_1]'V_1(t) + \sum_{t=1}^{t=T} K_3'(o)[I-(t-1)\hat{\imath}_3]'V_3(t) + \sum_{t=2}^{t=T} E'_2(t)V_2(t) - \sum_{t=2}^{t=T-1} D_3'(t)U_3(t) + \Big([I-\hat{\rho}_3][I-\hat{\delta}_3]S_3(o) - D_3(1)\Big)'U_3(4)$$

$$+\sum_{t=1}^{t=T-1} J_3'(t)W_3(t) + \Big(J_3(T) - S_3(T)\ 'W_3(T)\Big) - \Big([\hat{r}_3][I-\hat{\rho}_3][I-\hat{\delta}_3]S_3(o)\Big)'\Lambda_3(1) + \sum_{t=1}^{t=T-1} F_3'(t)\xi_3(t) + \Big(E_2(1) + S_2(o)\Big)'V_2(1) - \Big(C_1(o) + D_1(T) + [B_1]K_1(T) + [B_3]K_3(T)\Big)'\ U_1(T)$$

TRAINING POLICIES UNDER CONDITIONS OF TECHNICAL PROGRESS : A THEORETICAL TREATMENT

by C.C. von Weizsäcker

by

C. C. von Weizsäcker

Introduction

1 . One of the most important recent developments in growth economics has been the introduction of the time dimension into the analysis of capital stock. Economists formalized an idea which has, of course, been well known for a long time: new capital equipment is usually more effecient than old, not only because of the physical wear and tear going on all the time, but also because the former usually incorporates more advanced technical and scientific knowledge. Hence a machine or other piece of equipment depreciates in value even if it keeps all its initial physical characteristics through time. The formalization of this aspect is known as the "vintage" approach to growth

economics (1). Taking account of the superiority of new equipment over old usually implies that it is impossible to aggregate past investments in an unbiased way. Hence it is impossible to operate with the concept of an aggregate capital stock. The different vintages of machines remain in their own right, and the "wealth of a nation is to be described by a time series of past investment, rather than a single number, called the size of "capital stock". This is why these models are called vintage models. Looking at it from the point of view of technical progress rather than capital, this new approach also formalises the familiar fact that new knowledge has to be "embodied in new capital investment before it can become relevant to the economy. The vintage approach is thus at the same time the "embodied technical progress" approach. "Embodied technical progress" should not be confused with "induced technical progress". The latter refers to new knowledge produced (induced) by investments in physical or human capital; while the former refers to already existing new knowledge made relevant to the economy by investments. Embodied technical pregress may be induced technical progress, just as it may be autonomous; and induced technical progress can be of the embodied or of the disembodied variety.

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(1) Just a small selection of books and papers using this approach: L. Johansen: "Substitution versus Fixed Production Coefficients in the Theory of Economic Growth: A Synthesis in "Econometrica" 27, (1959). R.M. Solow: "Investment and Technical Progress", in Mathematical Methods in the Social Sciences", Stanford 1960. K. Arrow: "The Economic Consequences of Learning by Doing",in Review of Economic Studies" 29,(1962). N. Kaldor and J. Mirrlees: "A New Model of Economic Growth",in Review of Economic Studies, 29, (1962). E.S. Phelps: "Substitution, Fixed Proportions, Growth and Distribution", in "International Economic Review", 1963. R.M. Solow: "Capital Theory and the Rate of Return",Amsterdam 1963. I. Svennilson: "Economic Growth and Technical Progrêss" in "The Residual Factor and Economic Growth", OECD 1964. R.M. Solow, J.Tobin, C.C. von Weizsäcker, M. Yaari: "A Model of Fixed Capital without Substitution,"Review of Economic Studies" 1966. C.C. von Weizsäcker: "Zur ökonomischen Theorie des technischen Fortschritts", Göttingen, 1966 (forthcoming).

We may even say that in the case of induced technical progress new knowledge is induced by investments, by definition, whereas in the case of embodied technical progress investments are induced by technical progress: a new advance of knowledge will motivate entrepreneurs to buy machines which are constructed to apply in practice the newly found technical or scientific principle.

2. There is no reason why we should restrict the concept of embodied technical progress to the realm of physical capital. I therefore want to present a model of technical progress embodied in human capital. In pursuing the consequences of such a model a formalization of the concept of "éducation permanente" (Fourastié) will be found; which induces many contemporary discussions about appropriate educational policies in industrialised society.

The basic model

3. I shall construct a model which stresses the essential points, and is rather simple in all those details not essential for our topic. This is perhaps the most appropriate method for such an exploration of a new concept.

4. Consider an economy with a production function of the conventional type:

$$Y = F(K,A) \qquad (1)$$

where Y is the net social product, K the stock of physical capital and A the amount of labour, expressed in efficiency units.

λ is the number of years that a person stays in the educational or the production process. Let $N(v,t)$ be the number of persons in age class v who are in the labour force at time t. A person is in age class v when he reaches the minimum age necessary for entering the labour force at time v. Hence any person in age class v could enter the labour force at time v; but alternatively he could stay on at school. A person in age class v must enter the labour force at the latest at time $v + \lambda$. Let $F(v,t)$ be the average efficiency of a person of age class v at time t. The total labour force expressed in efficiency units, $A(t)$, is thus given by

$$A(t) = \int_{t-\lambda}^{t} N(v,t)\, F(v,t)\, dv$$

The question is what assumptions to make about $F(v,t)$. It is assumed that F depends directly on t and on the average level of training of the age class, $m(v,t)$. The simplest assumption is that

$$F(v,t) = e^{gt} f(m(v,t)).$$

The average efficiency is the product of an exponential function of t and a function, f, of the skill level of age class v, $m(v,t)$. The skill level is expressed in units of new years of training. In other words, if we compare two persons who were trained to the same extent up to last year, and if one person continued education this year whereas the other did not, their skill levels, m, will differ by one unit. Our central assumption is that a year of training in the past is less important today than a new year of training. Hence the skill level of a person is assumed to be a weighted sum of his past years of training, where the weight of the last period is unity and the weight decreases exponentially with the time between the present and the period in which the training took place. As time goes by, for any given past amount of training, this time increases,and hence the weight for the present skill level decreases exponentially, say at a rate $h > o$. This rate h will be called the rate of obsolescence. It has two components: (1) something which has been learned in the past will be increasingly forgotten; (2) something which has been learned in the past will become more and more obsolete because of new inventions and other new technical knowledge.

5. I think it is worthwhile to investigate the consequences of such an assumption. If we assume that the wage rates of different skill levels are proportionate to the efficiency levels, it is clear that the maximization of the discounted sum of wages induces behaviour which is efficient in the macro-economic sense.

Let $z_v(t)$ be the proportion of working time in period t which is used for learning (or being trained). Let $x_v(t)$ be the proportion of working time in period t which is used for earning money. Assuming that work consists only of learning and earning money, we know that $z_v(t)$ and $x_v(t)$ are numbers between zero and one, and that they add up to one:

$$0 \leq z_v(t),\ x_v(t) \leq 1 \qquad z_v(t) + x_v(t) = 1$$

or

$$x_v(t) = 1 - z_v(t)$$

The rate of change of the skill level, $\dot{m}(t,v)$, is then given by

$$\dot{m}(t,v) = z_v(t) - h\, m(t,v).$$

Money income in period t is given by

$$w\, f(m(t,v))\, e^{gt}\, (1 - z_v(t))$$

where w is a appropriately chosen constant, $f(m(t,v))e^{gt}$ expresses the efficiency level and $w\, f(m(t,v))e^{gt}$ is the wage rate.

$1 - z_v(t)$ is the amount of time in period t available for earning money. Assuming that direct training costs are zero which is not an essential assumption, and can be relaxed with no difficulty, the expression to be maximized is discounted life income, L_v, given by

$$L_v = w \int_v^{v+\lambda} e^{gt}\, f(m(t,v))(1-z_v(t))e^{-r(t-v)}\, dt$$

where r is the discount rate. In order to simplify notation we write

$$L_v = we^{gv} \int_0^{\lambda} e^{(g-r)t}\, f(m(t))(1-z(t))dt$$

dropping the index v under the integral sign, and redefining the variable t in such a way that t = o corresponds to time v. This can be done because v is kept constant in what follows. Thus the problem is to maximize $$\int_o^\lambda e^{(g-r)t} f(m(t))(1-z(t))dt$$

with respect to the functions m(t), z(t), such that

$$m(o) = o, \ \dot{m}(t) = z(t) - h\,m(t), \ o \le z(t) \le 1.$$

Solving the problem

This problem can be solved using the Maximum Principle of Pontryagin(1), or another method such as Dynamic Programming. From the Maximum Principle we infer that there exists a function p(t) such that the following two relations hold:

let $m^*(t)$, $z^*(t)$ be the optimal functions m(t), z (t);

1. then the inequality

$$H(z,t) = q(t)(1-z) + p(t)(z-h\,m) \le$$

$$H(z^*(t),t) = q(t)\quad(1-z^*) + p(t)\,(z^* - h\,m^*)$$

holds, where $q(t) = e^{(g-r)t} f(m^*(t))$, and z varies between o and 1.

2. $$\dot{p} - \dot{q} = h(p-q) + e^{(g-r)t}\left[h+r-g)f(m^*) - (1-h\,m^*)f'(m^*)\right]$$

These two relations have intuitive interpretations. Since q(t) is equal to the wage rate we can say it is the value of one unit of time if spent earning money. Similarly p(t) is the value of time if spent for training purposes. If $p > q$ then all time should be spent for training, i.e. $z^*(t)$ is equal to one. If $p < q$ then $z^*(t)$ is equal to zero. This leads to inequality (1.) Since the intuitive interpretation of (2.) is more complicated we will not enlarge on it here.[1]

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(1) L.S. Pontryagin, V.G. Boltyanski, R.V. Gamkrelidze, E.F. Mishchenko: "The Mathematical Theory of Optimal Processes", New York - London Sydney, 1963.

In order to give a more definite description of the optimal solution we have to make the following assumptions:

$$h + r - g > o$$

and

$$f(o) \geq o,\ f'(m) > o, \frac{f'(m)}{f(m)} \text{ is a decreasing function of } m.$$

The assumptions mean that f is always non-negative increasing with m, and that a modified law of decreasing marginal returns holds. Since the optimal $m(t)$ is everywhere non-negative and since it does not pay to increase m up to the last moment, we can infer that there exists t_o, with $o < t_o < \lambda$, such that $m^*(t_o) \geq m^*(t)$ for all t with $o \leq t \leq \lambda$. In other words, $m^*(t)$ reaches an interior maximum. Then we have $\dot{m}^*(t_o) = o$ and $\dot{m}^*(t_o - \epsilon) \geq o$, $\dot{m}^*(t_o + \epsilon) \leq o$ for sufficiently small $\epsilon > o$. If $m^*(t_o) > o$ this implies (dropping the sign) that

$$o = z(t_o) - h\, m(t_o) \text{ or } z(t_o) > o$$

that and therefore $p(t_o) \geq q(t_o)$. On the other hand, since $m(t) < \frac{1}{h}$ for all t, it follows from $\dot{m}(t_o +) \leq o$ that

$$o \geq z(t_o + \epsilon) - h\, m(t_o + \epsilon) \text{ or } z(t_o + \epsilon) < 1$$

and therefore that $p(t_o + \epsilon) \leq q(t_o + \epsilon)$ for any sufficiently small $\epsilon > o$. Thus $\dot{p}(t_o) - \dot{q}(t_o)$ must be non-positive which together with $p(t_o) \geq q(t_o)$ implies

$$(h+r-g)\, f(m(t_o)) - (1-h\, m(t_o)) f'(m(t_o)) \leq o$$

or

$$\frac{h+r-g}{1-h\, m(t_o)} \leq \frac{f'(m(t_o))}{f(m(t_o))}$$

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[1] For this intuitive interpretation see chapter 5 of my book on technical progress, op. cit.

But the left hand side increases and the right hand side decreases with m. Hence the maximum of $m(t)$, $m(t_0)$, is not greater than $\bar{m}$, the solution to the equation (1):

$$(h+r-g)\ f(\bar{m}) = (1 - h\ \bar{m})\quad f'(\bar{m})$$

Moreover, if $m(t_0) < \bar{m}$ we have $\dot{p}(t) - \dot{q}(t_0) < 0$ and hence $\dot{m}(t) > 0$ for $t = t_0 - \epsilon$ and $\dot{m}(t) < 0$ for $t = t + \epsilon$, if $\epsilon > 0$ is sufficiently small. In other words, $m(t_0)$ is a peaked maximum if it is less than $\bar{m}$.

We will now prove there exists no interior local minimum which is positive and not equal to $\bar{m}$. If such a minimum did exist, we would find a t, such that $0 < m(t_1) < \bar{m}$, $\dot{m}(t_1) = 0$, $\dot{m}(t_1 - \epsilon) \leq 0$, $\dot{m}(t_1 + \epsilon) \geq 0$ for sufficiently small positive ϵ, and hence $0 < z(t_1 < 1$, hence $p(t_1) = q(t_1)$ and $\dot{p}(t_1) - \dot{q}(t_1) \geq 0$ But this is a contradiction, since $p(t_1) = q(t_1)$ and $m(t_1) < m$ imply $\dot{p}(t_1) - \dot{q}(t_1) < 0$.

Since between two local maxima with value less than $\bar{m}$ there must exist a local minimum with value less than $\bar{m}$, we have proved that there exists at most one local maximum of value less than $\bar{m}$.

We can also prove that if $0 < m(t) < \bar{m}$, $z(t) = 0$ or $z(t) = 1$, or $m(t)$ is a local maximum. If $z(t) > 0$, we know that $p(t) \geq q(t)$. If $p(t) = q(t)$ we have $\dot{p}(t) - \dot{q}(t) < 0$ hence, as has been shown, $m(t)$ is a local maximum. If $p(t) > q(t)$, we have $z(t) = 1$.

6. What we have proved amounts to the following: there are three cases for $m(t)$:

Case I: $m(t) = 0$ for all t. No training pays most.

Case II: $m(t)$ has a unique (local and global) maximum, say at $t = t_0$. Then $m(t_0) \leq \bar{m}$, and hence $m(t) < \bar{m}$ for $t \neq t_0$, $m(t)$ is increasing for $t < t_0$, decreasing for $t > t_0$, then $z(t) = 1$ for $t < t_0$ and $z(t) = 0$ for $t > t_0$. The life span is divided into two periods: from zero to t_o all time is spent in education, from t_o to λ all time is spent earning moneay.

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(1) We assume that $\bar{m} \geq 0$ exists. If it does not, a similar argument shows that $m(t_0) = 0$; hence $m(t) = 0$ for all relevant t.

Case III: If there exists more than one maximum and no local interior positive minimum, and since $m(t)$ cannot become zero again once it is positive, there exists an interval, say from t_o to t_1, such that $m(t') = \max m(t)$ for $t_o \leq t' \leq t_1$ In addition, since there exists a unique maximum, if it is less than $\bar{m}$, we have $m(t') < \bar{m}$, for $t_o \leq t' \leq t_1$. Moreover, as in case II, $z(t) = 1$ for $t < t_o$, and $z(t) = o$ for $t > t_1$. Life is divided into three parts: from zero to t_o only education takes place; from t_o to t_1 part of each period is invested in further training, the other part is used for earning money and in the last interval from t_1 to λ the person concentrates on making money.

In modern society, parameters seem to indicate that case III is the most realistic for most people. The fact that until now further training of people who are already in the labour force is not very important, indicates that in most countries the educational system is not yet fully adapted to modern requirements.

Examples

In order to get a feeling for the quantities involved, let us specify the values of the parameters, and the type of the "production function", $f(m)$. We will then be able to compute the optimal lengths of the three periods involved. Let I be the first period, in which only education takes place; let II be the second period in which part of the time is spent in further education and part earning money; and let III be the last period in which no education takes place. We assume a total length of all three periods of 50 years say from the age of 15 to the age of 65. $\alpha = h\bar{m}$ is the proportion of time spent in education in period II. β is the ratio of education time during period II to the length of period I; that is

$$\beta = \frac{\alpha \text{ length of period II}}{\text{length of period I}}$$

β is an indicator of the proportion of the whole educational system that should be devoted to teaching people who are already in the labor force.

Table A refers to a linear efficiency function

$$f(m) = a + bm, \; a \geq o, b > o.$$

It indicates some typical values, for periods I, II, III, for given parameters h and r-g. The values are approximate.

Table B is the corresponding result in the case of an efficiency function of the general form

$$f(m) = a + b\sqrt{m} \quad a \geq o, b > o.$$

With the exception of the case h = 2%, r-g = o, the first line, for any given h and r-g, indicates the approximate maximum length of I, given the general structure of the function f(m)(1). The other lines refer to typical values of I, II, III, α and β for given h and r-g.

There are some interesting features of these figures to be noted:

(i) Keeping r-g constant, the length of all three periods is strongly influenced by h. Possibly the best single figure to indicate the educational structure is β. The value of β, for r-g = o, varies from zero for h = 2% up to 1.13 and 1.38 respectively for h = 5%.

(ii) Keeping h constant, the length of all three periods is strongly influenced by r-g. This can be seen, for example, in the case of h = 3%. The strong impact of the discount rate not only on the amount but also on the structure of education is perhaps more surprising than the influence of h.

(iii) For given h and r-g, as I increases, III decreases even faster; thus as I increases, II increases as well. This also implies that as the educational standard increases (reflected by an

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(1) This maximum length corresponds to the unrealistic case in which unskilled labour gets a wage of zero.

TABLE A						TABLE B				
Period			α	β		Period			α	β
I	II	III				I	II	III		
10	-	40	0	0	h = 2%	10	-	40	0	0
5	-	45	0	0	r-g = 0	5	-	45	0	0
16	4	30	30%	0.075	h = 2%	10	7	33	18%	0.13
10	-	40	0	0	r-g = 2%	4	6	40	7%	0.10
15	5	30	40%	0.13	h = 3%	10	10	30	27%	0.27
10	-	40	0	0	r-g = 0	4	6	40	11%	0.16
10	19	21	27%	0.51	h = 3%	7	23	20	20%	0.66
6	14	30	16%	0.37	r-g = 3%	4	21	25	11%	0.60
3	7	40	9%	0.21		1/3	20	30	1%	0.60
8	27	15	22%	0.74	h = 3%	4 1/2	30	16	13%	0.87
3	22	25	9%	0.66	r-g = 7%	1 1/3	29	20	4%	0.87
						1/3	29	21	1%	0.87
10	24	16	47%	1.13	h = 5%	7	29	14	33%	1.38
5	15	30	23%	0.69	r-g = 0	1 2/3	23	25	8%	1.15
						1/3	23	27	1 2/3%	1.15
7	31	12	33%	1.45	h = 5%	4	34	12	19%	1.7
3	26	21	14%	1.21	r-g = 5%	1/3	32	18	1 2/3%	1.6
1	19	30	5%	0.95						

increasing I), the coefficient β increases. But we have to admit that, in Table B at least, this relation between I and β is rather weak.

Possible extension of the model

7. This model is, of course, a considerable simplification of reality; but more complicated models can be constructed and evaluated numerically. These more complicated models should take account of the following features:

(a) The rate of obsolescence, h, is not a constant but a function of the age of the person.

(b) A person gains experience when he works in the labour force, and so his degree of skill also depends on the time he has spent in the labour force.

(c) After a fixed period of education an examination usually takes place. The income after the examination depends on whether or not the person passes the examination. This implies two modifications of f(m): firstly, f(m) becomes stochastic; secondly, f(m) ought to be approximated by a step function rather than by a continuous function, where the steps are given by the times when examinations take place.

(d) If we are interested in the optimal behaviour of an individual we have to take into account the fact that the capital market is not perfect, and hence the process of discounting becomes more complicated.

(e) A person has a preference function which also guides his choice between learning and working in the labour force. The criterion of life income maximization is therefore not realistic.

(f) There are usually considerable direct costs of education which we have to take into account.

Several other possible generalisations of this model could be listed. Most of these have not yet been investigated. I have done some preliminary research on points (a), (b), (f) and some other points not listed here.

Günter Kellner (Alfred-Weber-Institut, University of Heidelberg) is now working on some problems that arise when one tries to generalise the model. He and I hope to get far enough to use the model empirically and to draw practical conclusions from it for purposes of institutional and other reforms in the educational system.

PART V : COMMENTS

INTRODUCTION OF CONTROL CONCEPTS IN EDUCATIONAL PLANNING MODELS

by Paul Alper

> "It is...true that fertile ideas, conceptualisations and theories come, sometimes quite unexpectedly, from people of all philosophical persuasions."---Anatol Rapoport

Introduction

In order to discuss control concepts in educational planning models, a working definition is required. The phrase "working definition" is used because, although not all control engineers would subscribe to it, it will suffice for our purpose here.

A decade ago, an instructor defining a control system might have said that a control system was a servo-mechanism dealing with voltage inputs and mechanical outputs. Today this definition is completely misleading and wrong. It would be better to define a control system as one which can be taken from one state of nature to some other state of nature via an appropriate path by means of the application of automatic (or systematic) feedback processes. It is this working definition of control theory which should be born in mind for the following discussion of the relevance of control concepts to educational planning. (1)

One of the underlying concepts of "systems analysis" is that many seemingly different systems are, in fact, on an abstract basis, quite similar; that is, the various interrelationships of one system can be identified (at least, approximately) on a one to one basis with the interrelationships of a quite different system. The linear differential equations governing lumped electrical circuits are completely identical, for example, to those of lumped mechnical components, the partial differential equations of an electromagnetic wave guide are identical to the partial differential equations of a plucked string. The connection between control systems and educational systems is also quite strong, the great similarity lies in the fact that in both control systems and educational systems it is desired to take the system from one state of nature via an appropriate path to some other state of nature by means of the application of feedback processes. This abstract approach of looking at control systems has been extremely successful and thus it behooves the educational model-builder to make use of the immense amount of money and effort invested in control syytems in order to improve the comprehension and performance of educational systems

To a systems analyst, the essential difference between a control system and an educational system is not that the one involves voltages and torque and the other teachers and students; it is rather that the physics of the situation is better understood in the case of control systems than in the case of educational systems. The parameters and variables which are truly relevant for educational systems are as yet not known well enough in a quantitative sense. However, in both kinds of systems similar questions can be asked:

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(1) Note very carefully that the introduction of control concepts does not mean that one intends in any way to substitute a machine for a human being. More strongly, it is envisioned that the framework of control theory will aid the human being rather than replace him. The actual decision-making will still be in the hands of people; Various tools such as a desk calculator or a high-speed computer may be employed, but it is stressed that they are tools only and not ends in themselves.

1. Which objective functions are the proper ones for determining performance and at the same time are mathematically tractable enough for a meaningful answer to be obtained?
2. How sensitive are our answers with respect to certain assumptions?

. What mathematical tools can be employed in effecting solutions

4. How stable is the system and in what sense?
5. What are the ways to reduce uncertainty?

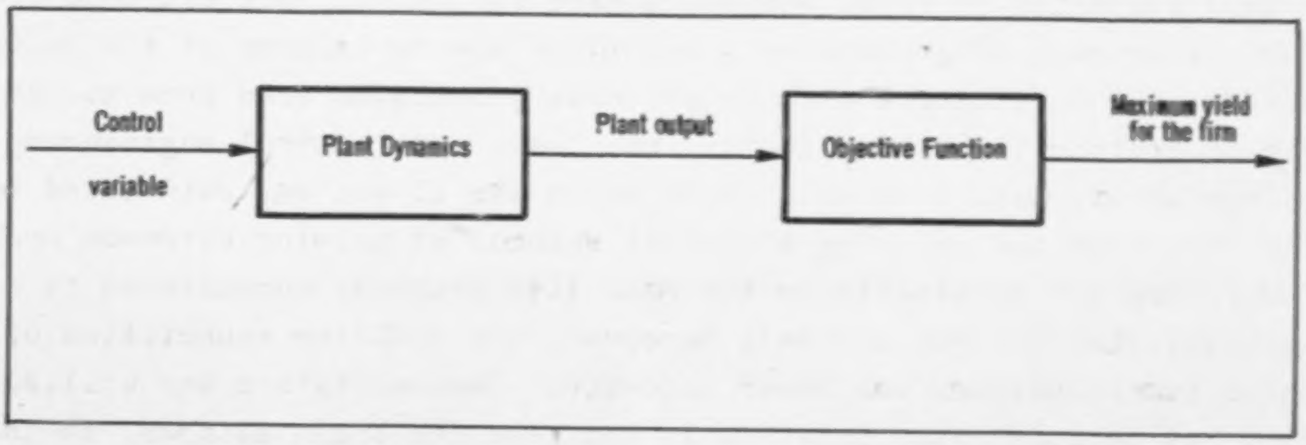
and so on.

In attempting to answer these and other questions, the controls engineer has exploited some very sophisticated results---results which could be great interest to the educational planner. For example, consider the paper of Prof. von Weizsäcker which makes use of Pontryagin's Maximal Principle, a mathematical technique of extreme importance in modern control theory because of its power in solving extremum problems with non-holonomic constraints (constraints which cannot be expressed by means of the equality sign). Note too that a different method suggested by Prof. von Weizsäcker for solving his extremum problem is dynamic programming-- a technique due to Bellman of the United States The development of both of these techniques (one from Russia, the other from the United States, the leaders in control engineering in the world) date from well after World War II and was stimulated by the fact that the existing classical methods of solving extremum problems were not applicable to the real life problems encountered in sophisticated control systems. Moreover, not just the enunciation of these two techniques but their successful implementation and utilization has occupied many individuals and research organizations, so that a vast body of literature has been built up on these subjects and one is able to discern when either technique is helpful in practice as well as in theory For example, Pontryagin's Maximal Principle can lead to two-point boundary value problems which are exceedingly difficult to solve, while dynamic programming suffers from the so-called "curse of dimensionality", meaning that for even a moderate number of state variables the computer memory (and this is true of even "next generation" computers) is swamped.

The conceptual superstructure which is offered by thinking in control engineering terms is quite powerful and general, and not at all limited (in principle, at any rate) to servo-mechanisms, chemical plants and the like. Indeed, at this very conference, several of the papers exhibit a control theory point of view, whether this is explicitly stated or not. That is to say, the models are not concerned with merely observing the phenomenon of education, but with influencing it as well so as to improve the performance of the educational system some way.

In fact, the comparison of a typical control problem of this decade concerning chemical plant optimization (as shown in Fig.1) with the educational optimization problem of Prof. von Weizsäcker [1] (as shown in Fig. 2) is quite revealing. One can immediately see the similarity of the two problems, and it is not surprising that the same mathematical tool, Pontryagin's Maximal Principle, can be effectively utilized in order to obtain a solution. Indeed, even the solutions are similar; that is, in the terminology of the control engineer, we often tend to obtain "bang-bang" types of solutions---full on (z=1, or all

Fig. 1. CHEMICAL PLANT OPTIMIZATION



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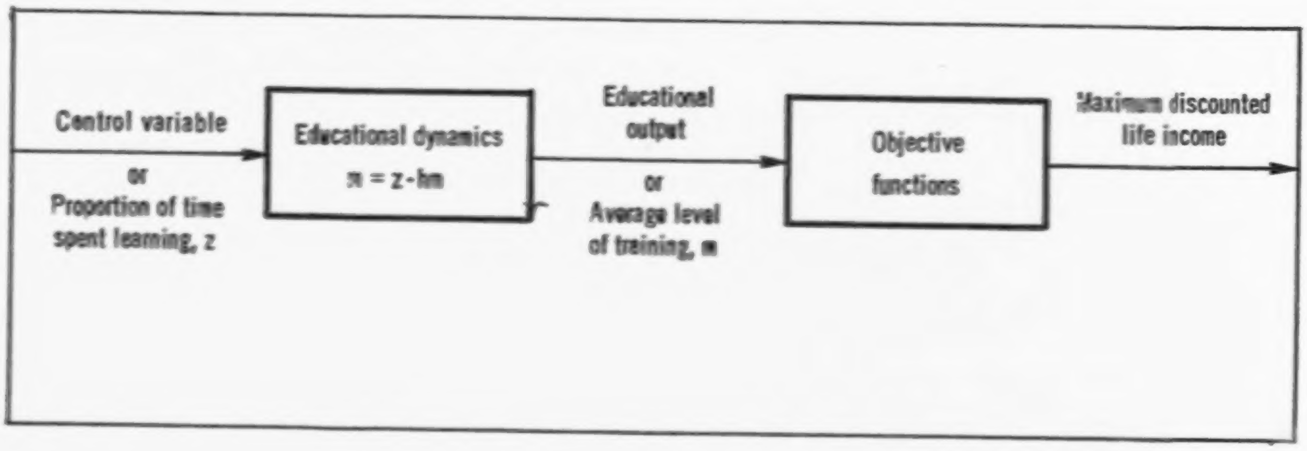
[1] Until the 1950 s, this problem could not be solved by classical means, such as the calculus of variations, because of the nonholonomic constraint on the proportion of time spent learning $(0 \leq z \leq 1)$.

learning; no earning) or full off (z=0, or all earning; no learning).

Fig. 2. EDUCATIONAL SYSTEM OPTIMIZATION


Control variable
or
Proportion of time
spent learning, z
Educational dynamics
ṁ = z-hm
Educational
output
or
Average level
of training, m
Objective
functions
Maximum discounted
life income

Nor is this the only paper dealing with controls. Both the papers of Dr. Smith and Mr. Armitage of England and of Dr. Swanfeld of Sweden make allusions to the desirability of incorporating within the educational planning models some sort of "feedback', the concept most closely associated with control theory. Indeed, the former paper devotes several pages to discussing the analogies to, and benefits from, control concepts with regard to educational models. It offers the idea of employing adaptive control features to their basic enrolment model so as to bring the forecast results closer to a target value. It also suggests a steering mechanism and control knobs for the basic model, in addition to pointing out the desirability (and pitfalls) of a dynamic control strategy.

Loosely speaking, the two above mentioned models involve, at present, a forecasting model. This means an open-loop or ballistic type of system (once started, there is no longer any way of modifying the results of perhaps some assumptions were not valid) as opposed to a proposed planning or closed-loop feedback model. In the first case, forecasting or open-loop, the situation would be as pictured in Fig. 3, while the second case, planning or closed-loop feedback control, would be as shown in Fig. 4. In the feedback case, the educational planner is afforded the opportunity of using updated and past information concerning student enrolment in order to change or modify the transition proportions. Operating in this feedback manner, one could compensate for deviations in the original analysis. For example, if it is desired that more females enter teaching than at present, then in order to obtain a desired goal, the transition proportion could be affected by increasing the monetary incentive so that teaching becomes more attractive. Similar incentives, both positive and negative (decreasing stipends or removing fringe benefits, for example), can be thought of.

Fig. 3 FORECASTING OR OPEN-LOOP EDUCATION SYSTEM

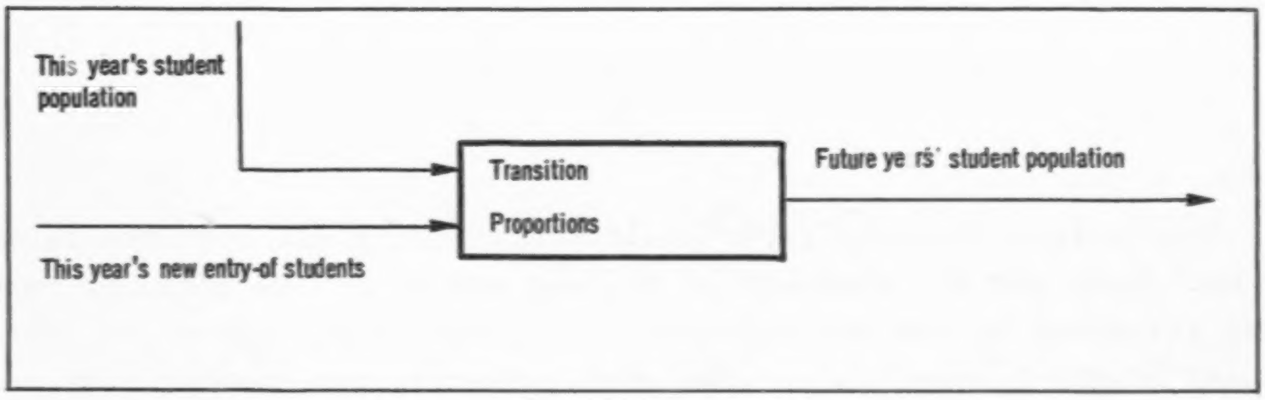
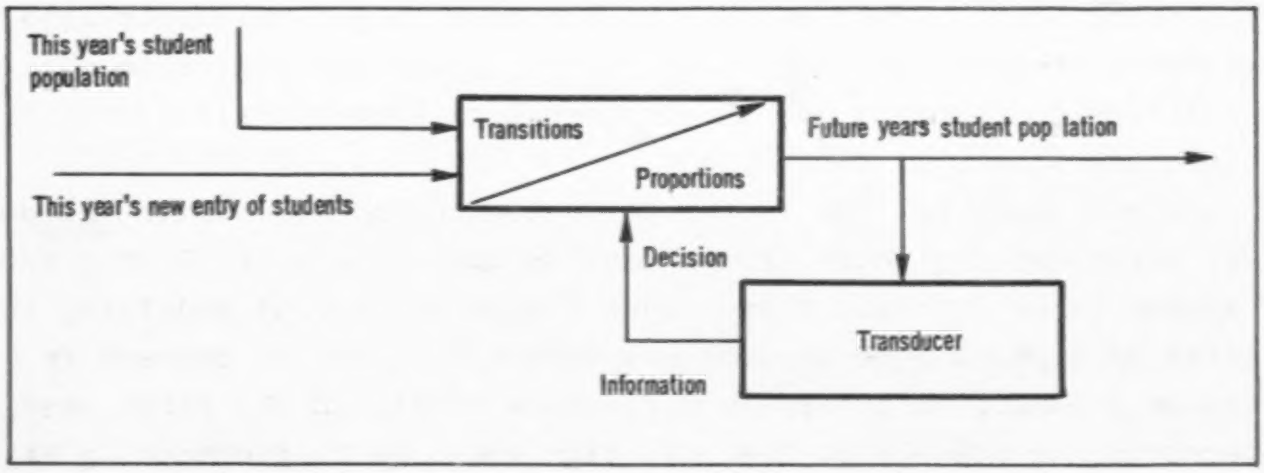

This year's student population
Transition
Proportions
Future years' student population
This year's new entry of students

Fig. 4 PLANNING OR CLOSED-LOOP EDUCATION SYSTEM


This year's student population
Transitions
Proportions
Future years' student population
This year's new entry of students
Decision
Transducer
Information

The above discussion is meant to bring out the fact that if one believes that mathematical models, abstractions and concepts can effectively be brought to bear on educational problems, then it should not be totally surprising that another discipline with all its own nomenclature and jargon can be extremely useful, both in a theoretical and practical way. One of the most important purposes of abstract mathematical models is to provide a conceptual superstructure which is independent of any particular frame of reference; and therefore it should not be too startling that another philosophical persuasion such as control theory, also based on abstract mathematical models, could possibly contribute very much to educational planning. Furthermore, because of the tremendous amount of research money invested in control problems and the huge number of people in the field (as compared to those planning educational models), intensive and far-reaching results have been produced which people in educational planning should be able to take advantage of and to utilize immediately and effectively.

With that in mind, I would like to suggest some ideas which I feel may be helpful.

Objective functions and extrema

The conceptual framework of control theory provides ways of formalizing some of the problems of educational systems as well as providing a means of "solving" the problems. For example, as the educational planner attempts to break away from models which merely forecast results to those models which can be controlled, it will become more and more necessary to stipulate performance criteria both with respect to the past data and the future evolution of the model, and to test the sensitivity of various assumptions as well as to determine the proper settings of the "infinitely variable knobs" referred to by Dr. Smith and Mr. Armitage(1). Any realistic objective function will necessarily involve a good deal of sophisticated mathematics, and with probably require some sort of electronic computer to yield an answer. Many of the same difficulties will arise in "solving" these problems as arise in solving large scale control problems:

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(1) Parenthetically it should be noted that even in theory, not all systems can be controlled in the sense of the working definition given in this paper.

1. Physically sensible criteria are often mathematically intractable.
2. Criteria which lead to analytic results often do not make sense physically.
3. Multiple peaks may exist in the performance function, coming the optimization procedure to reach a local rather than a global optimum.
4. The adjustment procedure can lead to instability; instead of finding the right settings, the entire system becomes unstable.
5. The adjustment procedure will converge to the wrong values.
6. The adjustment procedure can be too fast (unstable) or too slow (ineffective).
7. Two-point boundary value problems or "curse of dimensionality" (lack of storage space) may arise.

The educational model builder is therefore well advised to look to the control engineering literature in order that he may aid his computational and conceptual intuition.

In the engineering literature, the number of objective functions (performance criteria, pay-off functions, indices of performance) is legion, but one of the most popular in control theory is known as the "mean square error criterion". For convenience only let us apply it to the enrolment problem within the educational planning model framework. It could equally be applied in other areas, such as the supply and demand of teachers considered by Prof. Correa. We are trying to adjust the enrolment model so as to minimize the sum of the squares of the difference between a desired yearly vector distribution of students, $\bar{d}(t)$, and the actual distribution of students, $\bar{n}(t)$, over the range of time t=o to t=T. We therefore wish to minimize I, where

$$I = \sum_{t=o}^{T} (\bar{d}(t) - \bar{n}(t))' (\bar{d}(t) - \bar{n}(t)) \qquad (1)$$

or equivalently,

$$I = \sum_{t=o}^{T} \sum_{i=1}^{N} (d_i(t)-n_i(t))^2 \qquad (2)$$

where the subscript i indicates a component of the vector which has N components.

That is, we are seeking to adjust the various transition proportions in order that our model will produce the proper student outputs so that the objective function is mimimized. For example, suppose we want to select trends, a_k, in some of the transition proportions of the model such that the mean square deviation from recent years' data is minimized; then control theory indicates that the solution can be obtained by differentiating with respect to the trends, a_k, and setting

$$\frac{\partial I}{\partial a_k} = -2 \sum_{t=o}^{T} \sum_{i=1}^{N} (d_i(t)-n_i(t)) \frac{\partial n_i}{\partial a_k} = 0. \qquad (3)$$

This can be done using computor simulation, or even analytically if simple enough. Moreover, there is no conceptual reason to limit oneself to the mean square error criterion, and a more generalized criterion migh be

$$I = \sum_{t=o}^{T} \Big[(\bar{d}(t)-\bar{n}(t))'(\bar{d}(t)-\bar{n}(t)) \Big] m \qquad (4)$$

where m is any real number greater than zero.

One can also bring in the bottlenecks (mentioned in the paper of Dr. Smith and Mr. Armitage) by formulating the problem as one of calculus of variations, and, provided that the constraints are holonomic, the decision variables can be solved for.

For example, we assume that it is absolutely necessary that the actual distribution of students be exactly equal to the desired distribution of students in the last year of the plan, then

$$I = \sum_{t=o}^{T} \left[(\bar{d}(t)-\bar{n}(t))'(\bar{d}(t)-\bar{n}(t)) \right]^{m} : \lambda(\bar{d}(T)-\bar{n}(T)) \tag{5}$$

where λ is a Lagrange multiplier.

If the constraints are non-holonomic (e.g., the number of students studying medicine must be less than some prescribed figure due to lack of physical facilities), we can use Pontryagin's Maximal Principle, just as Prof. Von Weizsäcker has done for his maximization of discounted life income.

Space does not permit a proper exposition of the work already done on criteria; but I believe this approach will go a long way towards answering Professor Dressel's doubts which he expressed in his paper: "There seems to be no basis developed, or at least I found none, which permits determination of the amount of error which may be expected in planning projections. Do we simply assume that if the plans are made and put into effect the expected results will ensue? This would, indeed, be naive." Professor Dressel also states: "If the model is to be used for actual planning and attainment of certain economic goals then it seems to me essential to develop some measures of the errors inherent in the statistics or data which we insert in the model and some procedure for estimating the amount of error or difference between the outcomes and the goals which is to be expected upon projection into the future". In fact, control concepts can be employed not only to evaluate how effective the model is, but also to reduce the errors inherent in the statistics, a topic which is discussed under the heading of "Measurement Noise".

Identification and analysis of systems

It seems obvious that before one speaks of controlling the educational system one must first identify it.(1) Here too, the ideas of control theory would appear to be very helpful. For linear systems, one could envisage impulse response testing or its equivalent in the frequency domain, the transfer function of the system, to find out about an unknown system and to analyze what happens when subsystems are coupled together. For example,(2) when a linear subsystem of the educational system, let us say the secondary schools, receives an input from the primary schools, and produces an output which is the input to another subsystem of the educational system, say the universities, we would have a situation as shown in Fig. 5. Then, where $j = \sqrt{-1}$, $X(j\omega)$ is the Fourier transform of the output of the primary schools,

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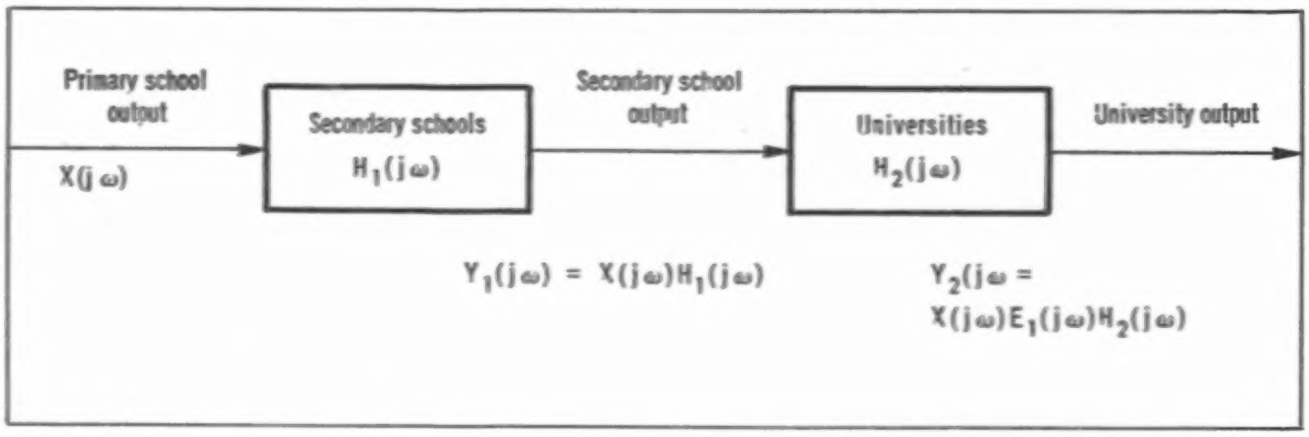
(1) Strange as it appears, this statement is not true in general, but for our purposes it may be assumed valid.

Fig. 5. COUPLING OF SUBSYSTEMS : THE FREQUENCY DOMAIN


Primary school output
$X(j\omega)$
Secondary schools
$H_1(j\omega)$
Secondary school output
Universities
$H_2(j\omega)$
University output
$Y_1(j\omega) = X(j\omega)H_1(j\omega)$
$Y_2(j\omega =$
$X(j\omega)E_1(j\omega)H_2(j\omega)$

$Y_1(j\omega)$ is the Fourier transform of the output of the secondary schools, $Y_2(j\omega)$ is the output of the universities, and $H_1(j\omega)$ and $H_2(j\omega)$ are the transfer functions of the secondary schools and universities respectively. Thus

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(2) For convenience only, the following discussion involves continuous rather than discrete time; all statements have counterparts involving discrete time.

$$Y_2(j\omega) = X(j\omega)H_1(j\omega)H_2(j\omega)=Y_1(j\omega)H_2(j\omega) \quad (6)$$

The corresponding expression in the time domain is far more complicated, and involves what is known as the "convolution integral". Moreover, great intuitive insight comes about when thinking in terms of the frequency domain. Furthermore, such things as time delay, which might for example occur because the effect of a decision is not manifested until years after the decision is made, can be quite nicely discussed in the frequency domain. Of course, the frequency domain concept is also the strongest tool for analyzing feedback.

The actual determination of H_1 (j) and H_2 (j) can be done in principle by several methods which need not necessarily involve a high speed computer. One can conceivably, in principle, at least, employ all the tricks of the trade, such as impulse response testing, square wave testing and even "white noise".

Perhaps even more important than the frequency domain are the more recent ideas of "state space" and "state variables" as means of describing systems. The state space approach analyses systems within the time domain, and is not limited to linear systems as is the frequency domain approach. Another great advantage of the state space method is that one can say a great deal about the solution without actually finding it. For example, one can find regions of stability using Lyapunov's Second Method - it would be possible to find out how much the system could be perturbed or disturbed and still return to its original equilibrium position and/or move to some other equilibrium position; that is, the so-called polystability of educational systems can be explored.

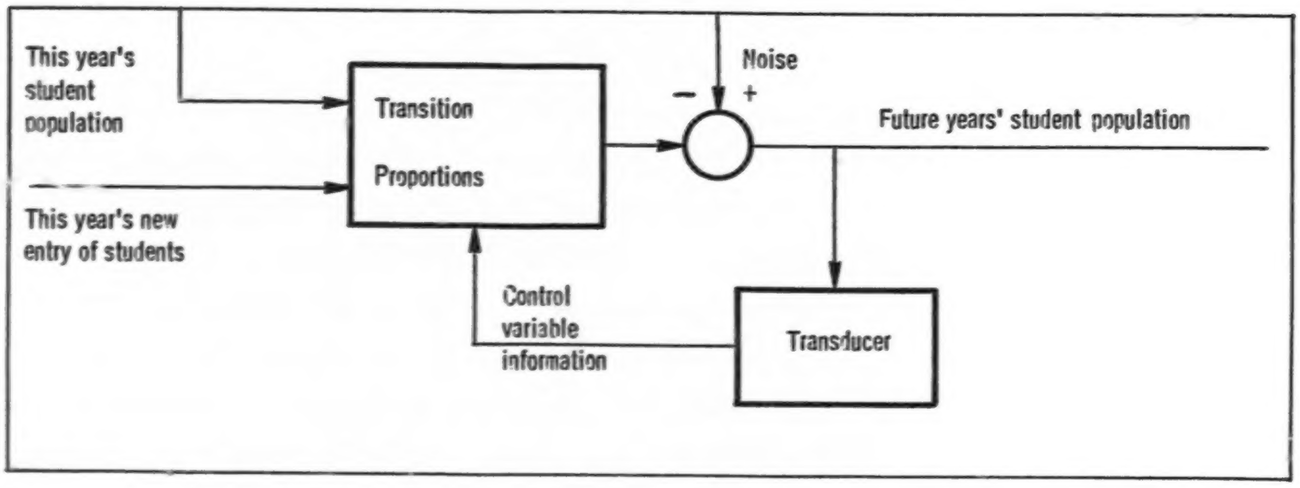
Unfortunately, a fuller discussion of state space and other techniques, such as Volterra series or the Describing Function for analyzing non-linear systems, is not possible at this point. However, it should be mentioned that non-linearities are no longer shunted aside as being intractable, and therefore it should not be beyond the realm of possibility for educational planning models in the future to break away from the confining chains of linear programming models when either the model, the objective function or both are non-linear.

Measurement noise

"Noise" can have many meanings to a control theorist, but for the purpose of this discussion attention is focused upon measurement noise, or any factor which prevents a precise and accurate determination of the parameters and variables of the model. Noise enters into educational planning models as quantification noise when, for example, data taken in one part of the country doesn't quite correspond to data in another part of the country (some universities are on a trimester basis while others have a semester basis), or when data for one type of variable doesn't correspond to that for another type of variable (data on teachers might be based on the calendar year while data on students might be based on the school year). Noise could also express the amount of uncertainty due to the fact that some variables or parameters are known only statistically. As an example, suppose that in Professor Correa's discussion on maximizing the benefits of a university curriculum, the benefits could not be agreed upon as a fixed number for each course but rather would be given by a distribution of values. Another example of noise would be the inaccurate determination of teacher supply because of uncertainties of immigration of teachers to the country and the return to teaching of middle-aged women who now have grown-up children. This case illustrates that noise would represent anything from the outside world which the planner does not know about, or which he cannot easily incorporate, or which tends to corrupt the results.

The way one usually conceptualizes the problem is hinted at in Fig. 6, where Fig. 4 is redrawn to bring in the noise aspect. The addition of noise or uncertainty makes the problem more difficult, but real-life problems are indeed more difficult. Fortunately, control theory has an extensive body of literature designed to reduce the effect of noise. Various networks, filters and algorithms exist so that the planner can optimize his model under the constraint of uncertainty. The problem of noise can be diminished by better data gathering, but it is something which model builders must not only learn to live with but learn to combat as well.

Fig. 6. PLANNING OR CLOSED-LOOP EDUCATION SYSTEM WITH NOISE


This year's student population
This year's new entry of students
Transition Proportions
Noise
−
+
Future years' student population
Control variable information
Transducer

Concluding remarks

This paper has attempted to point out that control concepts have a natural place not only in future educational planning models, but are in fact embodied in several of the papers at this conference. Control concepts can contribute greatly to the better understanding of the underlying structure and inter-relationships of the educational system, as well as providing a means of analysis. The crucial test, as always, remains in the proper implementation and utilization of the ideas.

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5

COMMENTS ON THE USE OF MATHEMATICAL MODELS IN EDUCATIONAL PLANNING

by Paul L. Dressel

Introductory comments

Planning is considered necessary when individual expectations are frustrated by the apparently meaningless interplay of an unregulated system and it tends to be advocated by those who have little or no control over the natural order of events. It is often resisted by those administrators who do have some measure of control in the "natural" system and who fear that a plan will limit their power to influence the course of events. When they do accept planning administrators often insist that much of it be done in secrecy. The resistance of administrators to planning and their desire to keep confidential the planning that is done contribute to the difficulties of anyone trying to build a planning model or a model of the planning mechanism.

One of the major problems encountered by model builders is the confusion of assumptions and goals. Unless the model makes clear which elements are variable and which must be accepted as fixed, little aid is given to the people actually responsible for administration of the plan. There are numerous examples of cases where decreasing pupil teacher ratios are assumed rather than built into the model as objectives.

Mathematical models have two major technical weaknesses. First; developing a model which strikes the proper balance between mathematical tractability and realistic representation of the system is extremely difficult. Second, the data required by a model is often so great that the time and effort spent on obtaining statistics may outweigh the benefits that the solution of the model might bring.

Critique of earlier educational planning models

After reading and in various degrees digesting several articles and books on mathematical models for educational planning, my first reaction concerned the gross oversimplification involved in some of the mathematical models and the apparent naivety in using them. As I read further I concluded that on the whole the accusation of naivety was ill-founded and that, indeed, the authors were well aware of the inadequacy of the models. I further became aware of some highly ingenious and adaptable models developed by individuals with a very keen insight into the mathematics involved.

Gradually, I found my experience as an educator and researcher taking over so that I became concerned with the assumptions and implications of the various models with regard to educations and society. I found myself also increasingly impressed with the importance of the work being done and with the difficulties involved. These difficulties seem to me to be of two types: first developing a logical mathematical structure which corresponds with reality, and, second obtaining the data required to bring a theoretical model to bear on the practical situation.

Any mathematical model or procedure depends upon certain assumptions or postulates. It also involves a set of operations. The relevance of a mathematical model to a practical problem is found precisely in the extent to which the assumptions and operations of the mathematical system correspond with the reality which is being studied. There are examples of a whole area of mathematics being developed out of the attempt to solve a physical problem, the tendency in the social sciences has been to adapt existing models or procedures to the social science problem. It is generally, therefore, not a difficult task to find out the nature of the mathematical assumptions and operations involved in the model. Analysis of the reality to which the model is to be applied is a much more formidable task. Almost inevitably that reality is so complicated that it must be grossly simplified in order that any readily manageable mathematical model will adequately represent it. Yet that very simplification may delimit or destroy the utility of the model.

Commenting on the success of the physical sciences since World War I, Prof. Bruce Crawford, in "Graduate Education Today" (p.218), recently remarked that: "They (the physical sciences) have progressed from the description of natural processes, through the understanding of them, into the control of them. The conceptual models, which at first merely reproduced the qualitative behavior of nature, have been successively refined to the point where the models approximate the real phenomena rather accurately; one may say that the theories of physical science have made contact with reality".

As an illustration, he points out that whereas the steam engine was developed largely as a result of intuition, empiricism, and trial and error, the atomic reactor and the laser and maser were developed as direct results of scientific theory. Model building and theory in the social sciences, by contrast, seem to me to be still at the descriptive stage. Can we ever hope to attain the accuracy of prediction by model and theory which the physical sciences have achieved?

In selecting a model, there must be some clarity about the use to which the model is to be put. Under some assumptions, a line segment may adequately represent the relationship between two variables but, if one wishes to investigate the value of one variable which maximizes the other, it is clear that, with such a model, the problem is trivial because one end or the other of the line is obviously the maximum point, depending entirely on the slope of the line.

In some problems a mathematical model assists in formalizing certain relationships so that, knowing one or more variables, the values of other variables can be determined. The entire range of values of the variables is utilized in building the model. Thus the relationships of height, weight, and other physical measurements can be worked out, and the extent of error in determining or predicting one or more measurements from a knowledge of others is readily determinable. With regression lines in time series one can determine from the available data some measure of closeness of fit or error and project into the future-assuming that, with continuance of the natural course of events, the same error limits will hold. Planning, however, involves projection in which certain parameters are changed, with the conscious intent that increased productivity will ensue. There seems to be no basis developed, or at least I found none, which permits determination of the amount of error which may be expected in planning projections. Do we simply assume that if the plans are made and put into effect the expected results will ensue? This would indeed, be naive.

Perhaps the underlying question here is really that of determining when a mathematical model is adequate. And the answer to this question may, again depend upon the use to which the model is to be put. If the model is to be used for actual planning and attainment of certain economic goals, then it seems to me essential to develop some measures of the errors inherent in the statistics or data which we insert in the model and some procedure for estimating the amount of error or difference between the outcomes and the goals which is to be expected upon projection into the future. For this purpose, we cannot know whether a mathematical model is satisfactory, nor can we know when we are making improvements in it unless we can either set up experimental situations for testing the model or collect data over a sufficient period of time to contrast expectations and outcomes.

If the model is being used primarily to attain some understanding of the interrelationship of a number of factors in economic development, thereby on one hand posing further questions for research, and on the other pointing to the need for more carefully defined and more extensive data collecting procedures, the matter of error may be of less significance. If the mathematical models are largely an attempt to develop a rational or logical approach to the planning, and educate individuals to analyze the long-run implications of a decision, before making that decision, fairly simple models may be adequate. If, as I suspect, in some cases the mathematical model is only seized upon by a theorist bemused with a planning problem but not actually engaged in planning, then it probably makes little difference whether the model in any sense approximates reality or not.

Mathematics offers a concise but highly abstract way of depicting relationships and of systematically exploring the cumulative effect of a series of operations. Thus matrices and possibly the special case of Markov chains are useful in input-output analysis. However, the educator who may know the extent and significance of the losses in students from one educational level to another, but who does not know any mathematics, may very well feel that an essentially simple idea has been made unreasonably complicated by the mathematical formulation.

Interests of educators and model builders also may differ somewhat, for whereas the model builder would like to obtain some fixed probabilities indicating continuance in school, the educator may be much more concerned with increasing retention (or decreasing wastage). Thus as the planner works to develop an applicable model, the educator works to destroy its applicability-although both may join in trying to increase the number of educated persons.

The role of assumptions

Phrases and sentences from here and there in what I have read suggest both a general awareness of the problem of assumptions and yet a tendency to shove the matter to one side. For example:

(i) In "Input-Output Relationships",(1) page 14: "... both assumptions are oversimplifications. Which of the two is better is something that can be tested only by experience, and this we do not yet possess".

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(1) References are fully identified at the end of the paper rather than being given as footnotes.

(ii) In "Econometric Models of Education", page 10: "... the purpose of the models is to aid us in the process of planning for education and for labor market policies, tacitly assuming that ways and means can be found to incline the population to seek the desirable education".

In other cases, what seem to me to be assumptions are not so stated. For example:

(i) In "Econometric Models of Education", page 12: "The ideal development of the educational system is one of regular growth parallel to the desired growth of the economy". I am not happy about this statement: I certainly do not accept it as a definition, and I would be inclined to quarrel with it as an assumption.

(ii) "Econometric Models of Education", page 27, has a section headed 'The Elimination of Surpluses of a Given Educational Attainment from the Labour Force'. I cannot help wondering why the issue of utilization of surpluses of a given educational attainment might not be equally significant to analyse. Do we ever really have surpluses of educated people or do we just fail to use them wisely?

(iii) In 'Economics of Human Resources", page 52: "The main function [of education] ..... is that of 'developing skills' ". One can admit that a function of education is that of developing skills, but whether it is the main function is something else again. So long as economists make such statements, they are likely to find that their materials are ignored or at least viewed with great suspicion by educators.

(iv) In "Economics of Human Resources", page 135: "The discovery of new ideas and knowledge is not a function of education, but of scientific investigation". I find this a particularly irritating statement because it seems to me that a major function of any university is the discovery of new ideas and knowledge, and that it is precisely in the discovery of new ideas and knowledge that the university makes one of its major contributions to the economy. Such statements seem to betray a limited conception of education. In the main, they may result from a desire to simplify the situation for analysis. One may wonder what can result from planning which starts from so limited a perspective.

Many of the immediately preceding remarks about assumptions are simply an indication of my disturbance at the limited conception of development adopted by economists. Are not increased literacy, responsible participation in society and in politics, identification with national goals, improvement in physical and mental health, and development of attitudes and values favorable to progress just as important as national income and industrial productivity? May not the ultimate economic return of such results be more significant than immediate gains in productivity? If so, how do we really measure the economic benefit of education? I distrust the manpower needs approach as an adequate basis for educational planning. I equally distrust the increased earnings of the educated as a basis for measuring the values of education. The residual approach, so far as I understand it, seems to me to involve so many unknowns that it is not really informative.

Education seems to me to be more in the nature of capital, for educated persons, like capital, can be used in many different ways. I see no equivalence and, excepting a limited number of fields requiring specialized and extended education, not even any very close relationship between education and vocation. I suspect that the specifically vocational education will increasingly be part-time on-the-job and continuing and recurrent rather than formal and limited in time or to one period of life. Thus the role of formal education may increasingly be to create acceptance of change and flexibility, and ability to continue one's education so as to bring about change and adapt to it. In the future, if not actually at present, deficiencies in general education rather than in vocational education may be the major cause of unemployability.

Some comments on specific articles and books

Although I shall inevitably exhibit my limited acquaintance with the literature on planning, I want to make comments on a number of the materials which I have read. These comments will, I think, make the generalizations which have preceded and which will follow more meaningful and perhaps more palatable. Let me comment first on Hector Correa's article, "Optimum Choice Between General and Vocational Education". Dr. Correa introduces a linear model which inevitably leads to 100 per cent vocational education or 100 per cent general education. The author himself notes (p.111): "For practical applications, models linearly expressed are usually oversimplified". This one certainly seems so in several respects. First, it is my observation that a great portion of vocational education depends upon or involves some general education. Since purely vocational education is often quickly outmoded in a rapidly developing technology, its probability of use may be zero as well as one, the latter being assumed by the author. Furthermore, highly specific vocational education may even condition a person against change and destroy his ability to change.

It is perhaps possible to imagine certain types of general education which have no utility, but general education, properly conceived, perhaps has maximal utility in the long run in that it can and should condition a person to flexibility, to further learning, and to ready acceptance of change. The development of engineering education is instructive in regard to this model. An increasing proportion of engineering education is being given over to the study of mathematics and basis sciences - essentially general education - because this pattern produces greater adaptability for further development in a rapidly changing technology. Dr. Correa also assumes that the probability of use or non-use of one subject is independent of the use or non-use of another subject. This seems to me to be altogether unrealistic and denies the validity of curriculums or programmes made up of interrelated courses.

Dr. Correa does include a variant model in which the number of vocationally trained persons is increased up to the point where the margin of income for vocationally trained persons is reduced to the level of income of persons having general education. He suggests a number of other non-linearities. It seems to me that until such non-linearities are investigated and developed there is little point in talking about optimum choice. I would suggest also that if such optimal choices are to be investigated, the choice should be between two or more alternatives which can be clearly differentiated.

I wish to comment briefly on a second article by Dr. Correa entitled,"Quantity versus Quality in Teacher Education". This again is a highly simplified linear model for maximizing the total product of education. While the basic issues raised by the author in this article are of concern, quality versus quantity is much more complicated than the model or even the article admits. There is no generally accepted definition of the results of education. It is clear, however, that in any economy some well educated persons will be required, and that their income will depend in part on the supply of such people in relation to the demand. Thus the relationship would seem to be non-linear. Furthermore, it is an oversimplification to assume that the quality of education depends only on the quality of teachers. In some respects, the number of levels of education or the structure and the relationship of these levels to economic needs, individual ability, and social needs are more important than teacher quality.

I should like to comment next on the article called "Quantitative Adaptation of Education to Accelerated Growth", again by Dr. Correa. The model used in this article is one used also in Dr. Correa's book, "The Economics of Human Resources", and in the OECD publication entitled, "Econometric Models of Education", containing papers by Tinbergen, Bos, and others. This model involves a series of linear equations - actually linear difference equations - with coefficients presumably determinable from actual economic and educational data. The equations relate economic growth and educational structure. They permit determination of the educational structure, assuming a given rate of economic growth, and they also permit examination of possible modes of transition from one educational structure to another required to support an increased rate of economic growth.

The model is based on the assumption that the number of people with secondary education must be proportional to the volume of production in the same time period. No doubt it is always possible to find a short enough time period so that this statement is true, but I wonder if it may not be that an increased rate of growth requires a higher proportion of persons at various educational levels simply because increasing productivity requires a higher level of technology and hence a higher level of education. It is of course possible to examine this possibility in the model, but it seems to me we need more actual knowledge about the necessary relationship between productivity and education before the model can be useful. A relationship based upon past data may not be the optimum relationship. One alternative suggested in this article is that a transition to a higher rate of growth might be accomplished by an alternative involving a reduction by ten per cent of the use of third-level manpower in production. This seems to me to be a highly unrealistic alternative even on a transition basis. I say this because it seems to me that any increase in productivity is likely to demand more rather than less third-level manpower. Despite such problems, this model seems to me to be an ingenious one capable of elaboration and refinement and highly promising for further exploration.

Next I should like to comment on the volume, "Economics of Human Resources", by Hector Correa. It seems to me that Economics of Human Resources" is a monumental initial effort to draw upon demography, psychology, physiology, education, and sociology in the attempt to develop a discipline of the economics of human resources. Much of the literature referred to in this volume is familiar ground to me. The range of research which was consulted is somewhat limited. It would take a major combined effort by repesentatives of the several relevant disciplines to review the literature and to undertake the delicate and difficult task of assessing its implications in the manner Dr. Correa has initiated. I believe that a major cooperative endeavor along these lines would be very valuable, both in forwarding the goals Dr. Correa has in mind, and in giving some direction and unity to research in the several disparate disciplines.

The article by Professor Richard Stone, "A Model of the Educational System", perhaps because it gives more overt attention to the problems of higher education, was particularly intriguing to me. Professor Stone points out that the model of this particular paper is a sub-model of a complex and economically important process. He further suggests that in attempting to develop models for economic and social activity it may be better to think in terms of linked models operating through exchanges of information rather than in terms of a single structure. The article itself is an elaboration of an educational model separately from the purely economic, with some suggestions as to how the two models might interact.

I was particularly impressed by Professor Stone's treatment of the "demand for places" and his comparison of this demand to a multistage epidemic in which changes in numbers infected depend both on the number presently infected and so liable to infect others and on the number not infected and so likely to catch the infection. I had never thought of the matter in quite this way, but it corresponds to an observation I made a few years ago - that the development of a new junior-community college in the state resulted in my own university getting more students from that immediate area than ever before. Apparently the disease caught on and infected a very large number of people very quickly! Professor Stone also makes the point that all forms of further training and retraining should be included among educational processes. He suggests that anticipated changes in educational technology can be introduced into his model. As one result of his epidemic approach to the demand for places, he notes that the past trends may be a poor guide to future changes (epidemics do run their course) and that different social classes are at very different stages of educational penetration, so that a thorough analysis may have to be broken down by classes or socio-economic groups.

In speaking of the alternative curriculums at the university, a branching process is suggested, and the point is made that changing economic prospects may be an important factor in the choices among these curriculums. Certainly this is borne out in my own experience and does, as Professor Stone suggests, provide a point for linking up the model of the educational system with the general model of the economy. I was also pleased with Professor Stone's comment that after compiling data to estimate parameters, it might well be that entirely new educational processes would have to be considered and a decision reached as to what processes would be used in the future. Professor Stone also comments that educational planning may need to be approached through a social cost-benefit analysis in which market values are a necessary but by no means sufficient ingredient. It may be appropriate to add that the consumption component of education should not be ignored, especially since its long-run economic significance has not yet been fully explored.

Professor Stone's comments are in accord with my own observations and prejudices. Perhaps for this reason it seems to me that his approach to educational planning is more realistic and adapts more readily to the unique character of education than those approaches which see education only as a means of meeting manpower needs. It is appropriate in concluding this section of comments on the writings of planners to use a quotation from "Input-Output Relationships". Commenting on one phase of the problem, the authors make the following statement (p.52):

> "... Constructing such a time-series with past statistics would not be impossible; but it would be a big undertaking and the accuracy of the results would still be doubtful because of the pitfalls inherent in reconciling different sets of data drawn up for different purposes and based on different classifications. It is useless to complain of such difficulties; one can only hope that statistics will gradually come to play a more important role in government and industrial policy than they do at present and so will improve in comprehensiveness, quality, and speed of publication...."

All those engaged in research and in planning can only say "amen" to that.

Some general comments and suggestions about educational planning

Planning at the macro level, while perhaps justifiable as a first approximation, seems to me to ignore social, psychological, and political factors.

Since we are dealing with people who are perhaps, on the whole, more governed by emotion than by logic, it seems to me that plans which ignore these factors are doomed to failure in advance.

Perhaps, too, although in a different sense than the economist may use the terms, macro level planning may be resisted because it appears to destroy uniqueness and individuality at the micro level. For example, the development of state-wide budgets for higher education in my own country is viewed with suspicion and concern by individual institutions and departments within those institutions because the specifics upon which the state-wide budget is based seem to destroy institutional and departmental initiative and doom all to mediocrity.

One of the major advantages of planning is the fact that it makes clear that resources are limited and that the decision to support one course of development is, in effect, a denial of alternatives. It seems to me, then, that one of the most effective approaches that planners can take is to develop alternative plans based on different sets of assumptions and goals so that those who must finally select and execute a plan can see more clearly the implications and the problems involved in a choice. In particular, the short- and long-range implications of a particular plan need to be explored and made clear. For example, it may be that for short-range economic growth emphasis on vocational rather than general education is preferable, but in the long run this decision may hamper or inhibit growth. Again, in the short-range programme of development, it may seem best to look to those educational institutions and those classes of society who already have positions of advantage and some degree of excellence as being most appropriate, but a more deliberate approach which equalizes opportunity may, in the long run, be not only more humane but economically more advantageous.

Even within a single institution the attempt to plan for the future arouses concern and even antagonism because:

(a) Those who have achieved a preferential position see their status threatened.

(b) Those with particular aspirations for the future would often rather risk continued uncertainty than possible denial of the aspirations.

(c) A plan with any degree of specificity is viewed by some as destructive of flexibility; they fear it may become self-validating. Administrators who operate on a free-wheeling basis of expediency and opportunism are very likely to be in this group. Related to this is the fear that a plan which denies certain possible developments will discourage donors who might support both these and other planned developments.

(d) Goals and assumptions are not clearly differentiated. Thus an apparently necessary assumption of a high student-teacher ratio, even though regarded as temporary and transitional, may be viewed as a goal which planners wish to achieve. An assumption about increasing use of educational technology such as television may provoke the same suspicion.

It seems to me that before the joint planning of economic and educational development can be succesful a much larger number of than at present will have to be brought to some understanding of what is involved in such planning and what penalties are involved in not planning. Perhaps educational programmes for the preparation of administrators, both educational and governmental, should give more attention to planning as a major function of administration. This may not be easy to accomplish, but I am intrigued by the possibility that such a highly readable document as "Educational Planning: Its Quantitative Aspects and Its Integration with Economic Planning" could be used for this purpose. It presents well the basic ideas and issues, and it avoids the mathematical formulations which might arouse distrust or even disgust.

Finally, I would say that the limited exposure which I have had to the work now being done in educational planning has been highly educational to me, and convinces me that more widespread attention to and support of these efforts should be forthcoming.

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